31^{st} International Physics Olympiad

Leicester, U.K.

Experimental Competition

Wednesday, July 12^{th}, 2000

Please read this first:

1. The time available is 2 ½ hours for each of the 2 experimental questions. Answers for your first question will be collected after 2 ½ hours.
2. Use only the pen issued in your back pack.
3. Use only the front side of the sheets of paper provided. Do not use the side marked with a cross.
4. Each question should be answered on separate sheets of paper.
5. For each question, in addition to the ***blank writing sheets*** where you may write, there is an ***answer sheet*** where you *must* summarise the results you have obtained. Numerical results should be written with as many digits as are appropriate to the given data. Do not forget to state the units
6. Write on the blank sheets of paper the results of all your measurements and whatever else you consider is required for the solution of the question and that you wish to be marked. However you should use mainly equations, numbers, symbols, graphs and diagrams. Please use *as little text as possible*.
7. *It is absolutely essential* that you enter in the boxes at the top of each sheet of paper used your ***Country*** and your student number (***Student No***.). In addition, on the blank sheets of paper used for each question, you should enter the number of the question ***(Question No.)***, the progressive number of each sheet ***(Page No.)*** and the total number of blank sheets that you have used and wish to be marked for each question ***(Total No. of pages)***. It is also helpful to write the question number and the section label of the part you are answering at the beginning of each sheet of writing paper. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large cross through the whole sheet and do not include it in your numbering.
8. When you have finished, arrange all sheets *in proper order* (for *each* question put answer sheets first, then used sheets in order, followed by the sheets you do not wish to be marked. Put unused sheets and the printed question at the bottom). Place the papers for each question inside the envelope labelled with the appropriate question number, and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

CDROM SPECTROMETER

In this experiment, you are NOT expected to indicate uncertainties in your measurements.

The aim is to produce a graph showing how the conductance* of a light-dependent resistor (LDR) varies with wavelength across the visible spectrum.

*conductance G = 1/resistance (units: siemens, $1\ S = 1\ \Omega^{-1}$)

There are five parts to this experiment:

- *Using a concave reflection grating (made from a strip of CDROM) to produce a focused first order spectrum of the light from bulb A (12 V 50W tungsten filament).*
- *Measuring and plotting the conductance of the LDR against wavelength as it is scanned through this first order spectrum.*
- *Showing that the filament in bulb A behaves approximately as an ideal black body.*
- *Finding the temperature of the filament in bulb A when it is connected to the 12 V supply.*
- *Correcting the graph of conductance against wavelength to take account of the energy distribution within the spectrum of light emitted by bulb A.*

Precautions

- *Beware of hot surfaces.*
- ***Bulb B should not be connected to any potential difference greater than 2.0 V.***
- *Do not use the multimeter on its resistance settings in any live circuit.*

Procedure

(a) The apparatus shown in Figure 1 has been set up so that light from bulb A falls normally on the curved grating and the LDR has been positioned in the focused **first order** spectrum. Move the LDR through this **first-order** spectrum and observe how its resistance (*measured by multimeter X*) changes with position.

(b) (i) Measure and record the resistance R of the LDR at different positions within this first-order spectrum. Record your data in the blank table provided.

(ii) Plot a graph of the conductance G of the LDR against wavelength λ using the graph paper provided.

Note The angle θ between the direction of light of wavelength λ in the first-order spectrum and that of the white light reflected from the grating (see Figure 1) is given by:

$\sin\theta = \lambda / d$ where d is the separation of lines in the grating.

The grating has 620 lines per mm.

The graph plotted in (b)(ii) does not represent the sensitivity of the LDR to different wavelengths correctly as the emission characteristics of bulb A have not been taken into account. These characteristics are investigated in parts (c) and (d) leading to a corrected curve plotted in part (e).

- ***Note for part (c) that three multimeters are connected as ammeters. These should NOT be adjusted or moved. Use the fourth multimeter (labelled X) for all voltage measurements.***

(c) If the filament of a 50 W bulb acts as a black-body radiator it can be shown that the potential difference V across it should be related to the current I through it by the expression:

$$V^3 = CI^5 \text{ where } C \text{ is a constant.}$$

Measure corresponding values of V and I for bulb A (in the can). *The ammeter is already connected and should not be adjusted.*

(i) Record your data and any calculated values in the table provided on the answer sheet.

(ii) Plot a suitable graph to show that the filament acts as a black-body radiator on the graph paper provided.

(d) To correct the graph in (b)(ii) we need to know the working temperature of the tungsten filament in bulb A. This can be found from the variation of filament resistance with temperature.

- **You are provided with a graph of tungsten resistivity ($\mu\,\Omega$ cm) against temperature (K).**

If the resistance of the filament in bulb A can be found at a known temperature then its temperature when run from the 12 V supply can be found from its resistance at that operating potential difference. Unfortunately its resistance at room temperature is too small to be measured accurately with this apparatus. However, you are provided with a second smaller bulb, C, which has a larger, *measurable* resistance at room temperature. Bulb C can be used as an intermediary by following the procedure described below. You are also provided with a second 12V 50W bulb (B) identical to bulb A. Bulbs B and C are mounted on the board provided and connected as shown in Figure 2.

(i) Measure the resistance of bulb C when it is unlit at room temperature (*use multimeter X*, and take room temperature to be 300 K). Record this resistance R_{C1} on the answer sheet.

i. Use the circuit shown in Figure 2 to compare the filaments of bulbs B and C. Use the variable resistor to vary the current through bulb C until you can see that overlapping filaments are at the same temperature. If the small filament is cooler than the larger one it appears as a thin black loop. Measure the resistances of bulbs B and C when this condition has been reached and record their values, R_{C2} and R_B, on the answer sheet. *Remember, the ammeters are already connected.*

(iii) Use the graph of resistivity against temperature (supplied) to work out the temperature of the filaments of B and C when they are matched. Record this temperature, T_{2V}, on the answer sheet.

(iv) Measure the resistance of the filament in bulb A (in the can) when it is connected to the 12 V a.c. supply. *Once again the ammeter is already connected and should not be adjusted*. Record this value, R_{12V} on the answer sheet.

(v) Use the values for the resistance of bulb A at 2 V and 12 V and its temperature at 2 V to work out its temperature when run from the 12 V supply. Record this temperature, T_{12V} in the table on the answer sheet.

- **You are provided with graphs that give the relative intensity of radiation from a black-body radiator (Planck curves) at 2000 K, 2250 K, 2500 K, 2750 K, 3000 K and 3250 K.**

(e) Use these graphs and the result from (d)(v) to plot a corrected graph of LDR conductance (arbitrary units) versus wavelength using the graph paper provided. Assume that the conductance of the LDR at any wavelength is directly proportional to the intensity of radiation at that wavelength (This assumption is reasonable at the low intensities falling on the LDR in this experiment). Assume also that the grating diffracts light equally to all parts of the first order spectrum.

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Figure 1 - Experimental arrangement for (a)

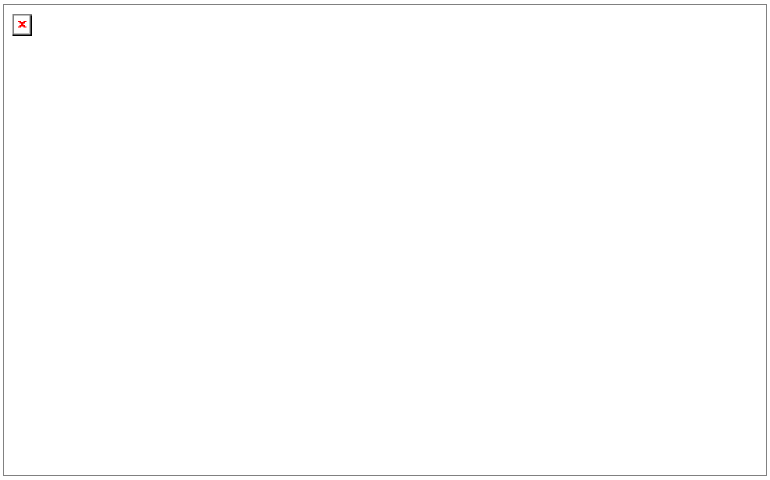


Figure 1: Detail - the grating:

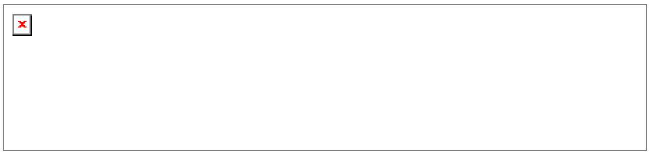


Figure 1: Detail - LDR and Multimeter:

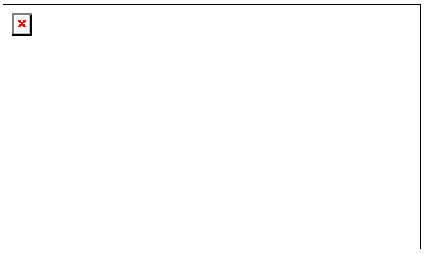
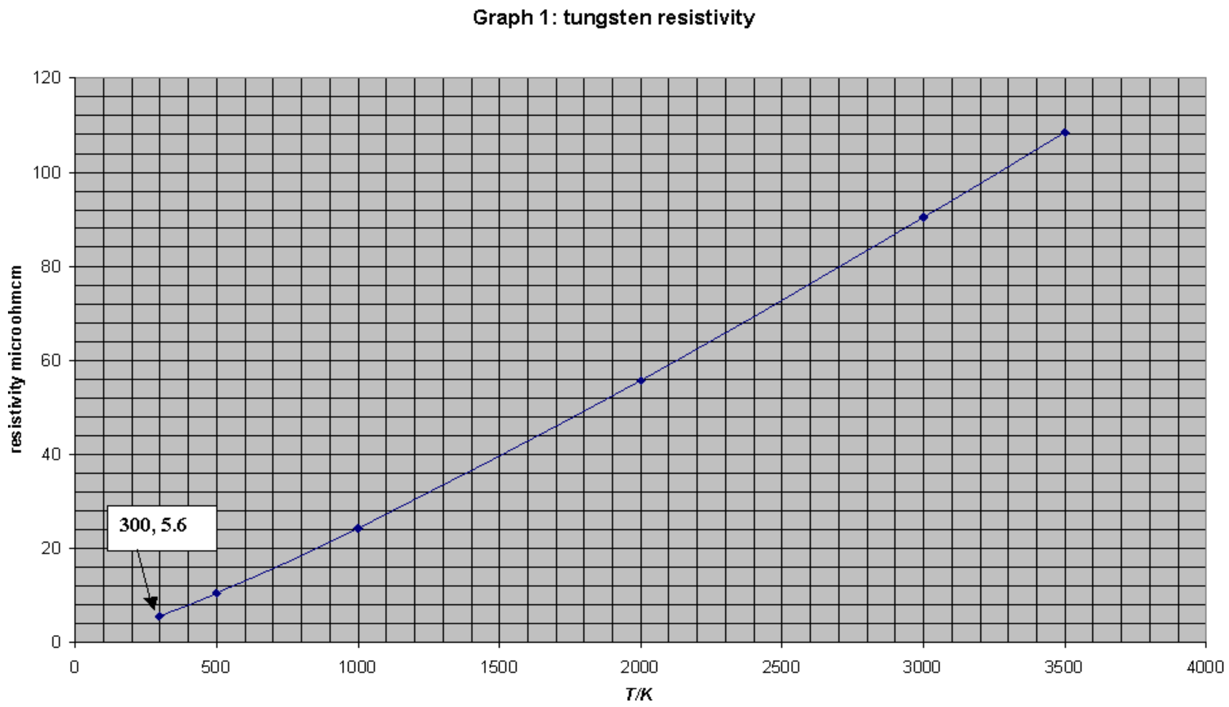


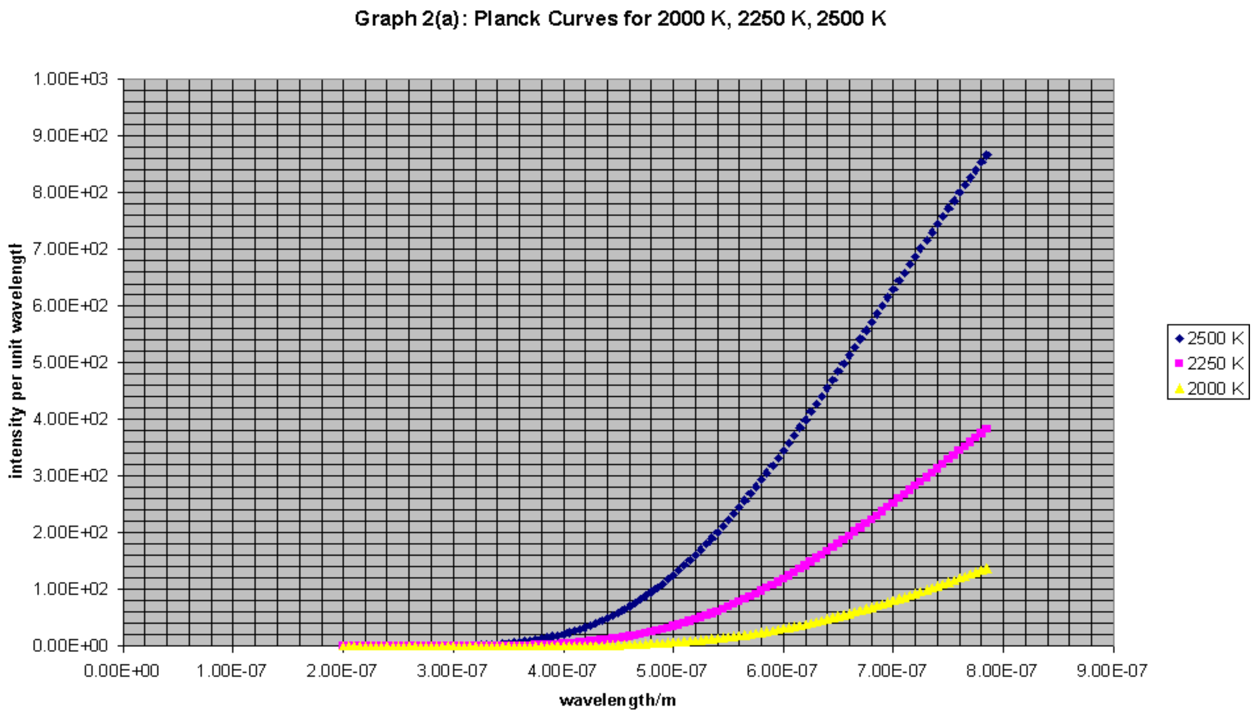
Figure 2

Note that this diagram does not show meters

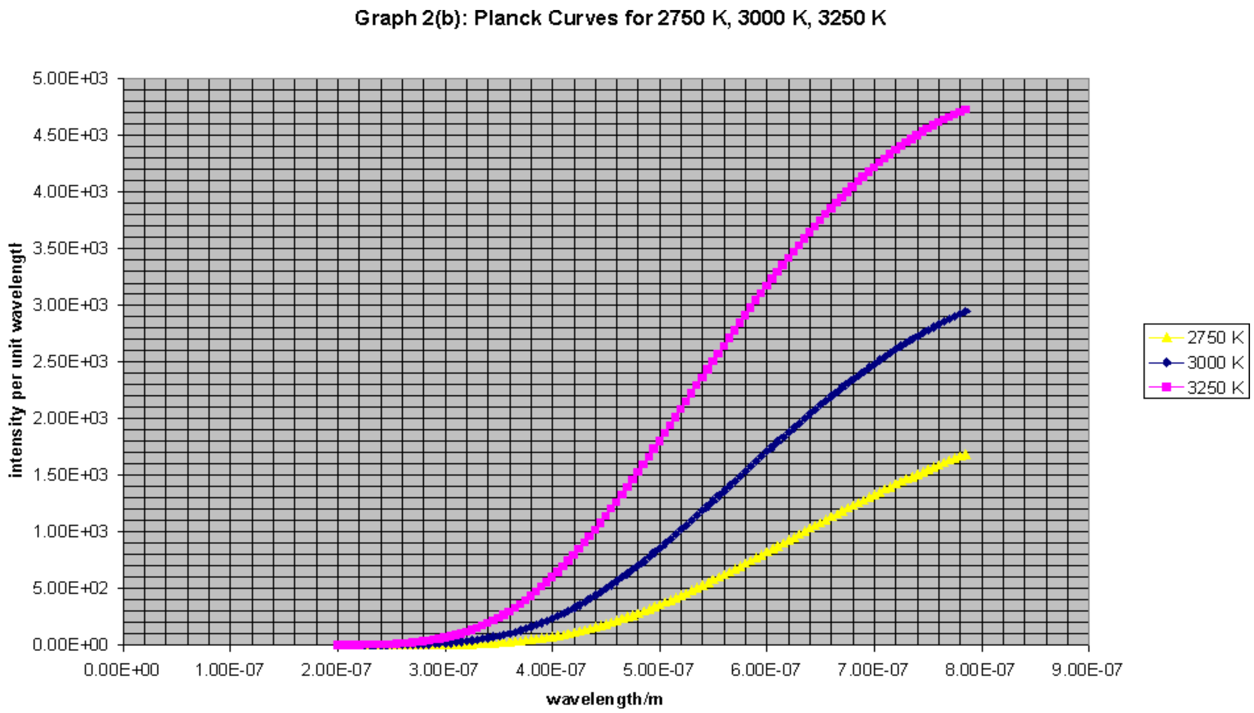
Graph 1



Graph 2a



Graph 2b


Graph 2(b): Planck Curves for 2750 K, 3000 K, 3250 K
intensity per unit wavelength
wavelength/m
5.00E+03
4.50E+03
4.00E+03
3.50E+03
3.00E+03
2.50E+03
2.00E+03
1.50E+03
1.00E+03
5.00E+02
0.00E+00
0.00E+00
1.00E-07
2.00E-07
3.00E-07
4.00E-07
5.00E-07
6.00E-07
7.00E-07
8.00E-07
9.00E-07
2750 K
3000 K
3250 K

The Magnetic Puck

July 2000

2.5 hours

In this experiment you ARE expected to indicate uncertainties in your measurements, results and graphs

Aim

To investigate the forces on a puck when it slides down the slope.

Warning

*Do **not touch** the circular flat faces of the puck or the paper surface of the slope with your hands. Use the glove provided. The faces have different coloured paper stickers for convenience but the frictional characteristics of the paper faces may be assumed to be the same.*

Timing

The sensors underneath the track trigger electronic gates in the box and the green LED will light when the puck is between the sensors. The multimeter measures the potential difference across a capacitor, which is connected to a constant-current source (whose current is proportional to the voltage of the battery) whilst the green light is on. The reading of the multimeter is therefore a measure of the time during which the puck is between the sensors. This reading can give a value for the speed of the puck in arbitrary units.

Operating the timer

i) Press and hold down the black push button on the side of the box. This switches the electronics on.

ii) If the green light goes on slide the puck (light face up) past the lower sensor. The green light should go off.

iii) The potential difference across the capacitor can be reduced to zero before the puck is released by pressing the red button for at least 10s.

iv) The battery potential difference can be measured by connecting the multimeter across the terminals marked with the cell symbol.

Definitions

(i) A moving body sliding down an inclined plane experiences a tangential retarding force F and a normal reaction N. Define

$$\mu = \frac{F}{N}$$

(ii) When the retarding force is due to friction alone, x equals m_s and is called the dynamic coefficient of friction for the surface. It is independent of speed.

(iii) When the blue (dark) side is in contact with the plane define

$$\xi_d = \frac{F_d}{N}$$

where the tangential force F_d is partly due to the surface friction and partly due to magnetic effects.

(iv) The variable x_{ds} which gives the magnetic effects only is defined by

$$\xi_{ds} = \xi_d - \mu_s$$

Important hints and advice

(i) You will find it helpful initially to investigate the behaviour of the puck qualitatively.

(ii) Think about the physics before you do a quantitative investigation. Remember to use graphical presentation where possible.

(iii) Do not attempt to take too many experimental readings unless you have plenty of time.

(iv) You are measuring the potential difference across an electrolytic capacitor. This does not behave quite like a simple air capacitor. Slow leakage of charge is normal and the potential difference will not remain completely steady.

(v) You are given one puck and one 9.0 V battery. Conserve the battery! The constant current filling the capacitor is proportional to the battery potential difference. It is therefore advisable to monitor the battery potential difference. In addition, the sensors may not be reliable if the potential difference of the battery falls below 8.4 V. If this happens, ask for another battery.

(vi) Your answer pack contains 4 sides of graph paper only. You will not be given further sheets. You may keep the puck at the end of your experiment.

(vii) If you have trouble operating the multimeters ask an invigilator.

Data

- Weight of puck = $5.84´10^{-2}$ N

- The voltmeter reading indicates the time of travel of the puck. When the potential difference of the battery is 9.0 V then 1V corresponds to 0.213 s

- Distance between sensors = 0.294 m

Experiment

Using only the apparatus provided investigate how x_{ds} depends on the speed v_q of the puck for track inclinations q to the horizontal.

State on the answer sheet the algebraic equations/relations used in analysing your results and in plotting your graphs.

Suggest a quantitative model to explain your results. Use the data which you collect to justify your model.

Theoretical Problem 1

Part A

A bungee jumper is attached to one end of a long elastic rope. The other end of the elastic rope is fixed to a high bridge. The jumper steps off the bridge and falls, from rest, towards the river below. He does not hit the water. The mass of the jumper is m, the unstretched length of the rope is L, the rope has a force constant (force to produce 1 m extension) of k and the gravitational field strength is g.

You may assume that

- the jumper can be regarded as a point mass m attached to the end of the rope,
- the mass of the rope is negligible compared to m,
- the rope obeys Hooke's law,
- air resistance can be ignored throughout the fall of the jumper.

Obtain expressions for the following and insert on the answer sheet:

- the distance y dropped by the jumper before coming instantaneously to rest for the first time,
- the maximum speed υ attained by the jumper during this drop,
- the time t taken during the drop before coming to rest for the first time.

Part B

A heat engine operates between two identical bodies at different temperatures T_A and T_B ($T_A > T_B$), with each body having mass m and constant specific heat capacity s. The bodies remain at constant pressure and undergo no change of phase.

1. Showing full working, obtain an expression for the final temperature T_0 attained by the two bodies A and B if the heat engine extracts from the system the maximum amount of mechanical work that is theoretically possible.

   Write your expression for the final temperature T_0 on the answer sheet.

2. Hence, obtain and write on the answer sheet an expression for this maximum amount of work available.

   The heat engine operates between two tanks of water each of volume 2.50 m^3. One tank is at 350 K and the other is at 300 K.

3. Calculate the maximum amount of mechanical energy obtainable. Insert the value on the answer sheet.

Specific heat capacity of water = 4.19×10^3 J kg^{-1} K^{-1}

Density of water = 1.00×10^3 kg m^{-3}

Part C

It is assumed that when the earth was formed the isotopes ^{238}U and ^{235}U were present but not their decay products. The decays of ^{238}U and ^{235}U are used to establish the age of the earth, *T*.

a. The isotope ^{238}U decays with a half-life of 4.50×10^9 years. The decay products in the resulting radioactive series have half-lives short compared to this; to a first approximation their existence can be ignored. The decay series terminates in the stable lead isotope ^{206}Pb

   Obtain and insert on the answer sheet an expression for the number of ^{206}Pb atoms, denoted ^{206}n, produced by radioactive decay with time t, in terms of the present number of ^{238}U atoms, denoted ^{238}N, and the half- life time of ^{238}U. (You may find it helpful to work in units of 10^9 years.)

b. Similarly, ^{235}U decays with a half-life of 0.710×10^9 years through a series of shorter half-life products to give the stable isotope ^{207}Pb.

   Write down on the answer sheet an equation relating ^{207}n to ^{235}N and the half-life of ^{235}U.

c. A uranium ore, mixed with a lead ore, is analysed with a mass spectrometer. The relative concentrations of the three lead isotopes ^{204}Pb, ^{206}Pb and ^{207}Pb are measured and the number of atoms are found to be in the ratios 1.00 : 29.6 : 22.6 respectively. The isotope ^{204}Pb is used for reference as it is not of radioactive origin. Analysing a pure lead ore gives ratios of 1.00 : 17.9 : 15.5.

   Given that the ratio ^{238}N : ^{235}N is 137 : 1, derive and insert on the answer sheet an equation involving *T*.

d. Assume that *T* is much greater than the half lives of both uranium isotopes and hence obtain an approximate value for *T*.

e. This approximate value is clearly not significantly greater than the longer half life, but can be used to obtain a much more accurate value for *T*.
   Hence, or otherwise, estimate a value for the age of the earth correct to within 2%.

Part D

Charge *Q* is uniformly distributed *in vacuo* throughout a spherical volume of radius *R*.

a. Derive expressions for the electric field strength at distance r from the centre of the sphere for $r \leq R$ and $r > R$.

b. Obtain an expression for the total electric energy associated with this distribution of charge.

Insert your answers to (a) and (b) on the answer sheet.

Part E

A circular ring of thin copper wire is set rotating about a vertical diameter at a point within the Earth's magnetic field. The magnetic flux density of the Earth's magnetic field at this point is 44.5 μT directed at an angle of 64^{o} below the horizontal. Given that the density of copper is 8.90×10^{3} kg m^{-3} and its resistivity is 1.70×10^{-8} Ω m, calculate how long it will take for the angular velocity of the ring to halve. Show the steps of your working and insert the value of the time on the answer sheet. This time is much longer than the time for one revolution.

You may assume that the frictional effects of the supports and air are negligible, and for the purposes of this question you should ignore self-inductance effects, although these would not be negligible.

Theoretical Problem 2

a. A cathode ray tube (CRT), consisting of an electron gun and a screen, is placed within a uniform constant magnetic field of magnitude **B** such that the magnetic field is parallel to the beam axis of the gun, as shown in figure 2.1.

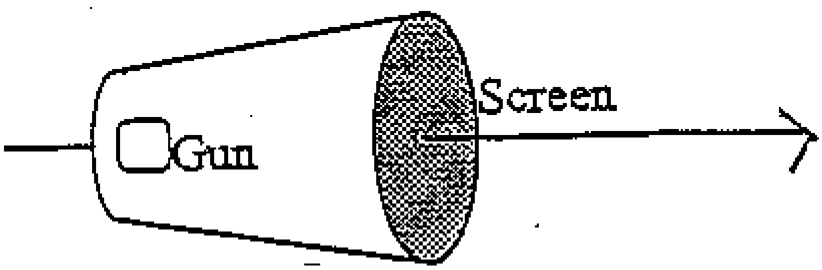


Figure 2.1

The electron beam emerges from the anode of the electron gun on the axis, but with a divergence of up to 5° from the axis, as illustrated in figure 2.2. In general a diffuse spot is produced on the screen, but for certain values of the magnetic field a sharply focused spot is obtained.

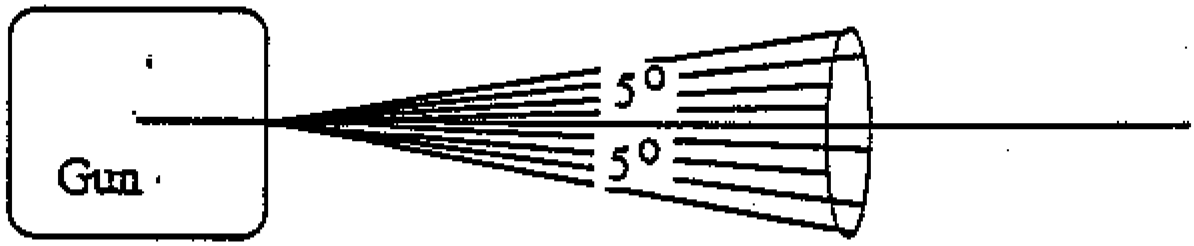


Figure 2.2

By considering the motion of an electron initially moving at an angle β (where $0 \leq \beta \leq 5°$) to the axis as it leaves the electron gun, and considering the components of its motion parallel and perpendicular to the axis, derive an expression for the charge to mass ratio e/m for the electron in terms of the following quantities:

- the smallest magnetic field for which a focused spot is obtained,
- the accelerating potential difference across the electron gun V (note that $V < 2$ kV),
- D, the distance between the anode and the screen.

Write your expression in the box provided in section 2a of the answer sheet.

b. Consider another method of evaluating the charge to mass ratio of the electron. The arrangement is shown from a side view and in plan view (from above) in figure 2.3, with the direction of the magnetic field marked **B**. Within this uniform magnetic field **B** are placed two brass circular plates of radius ρ which are separated by a very small distance *t*. A potential difference *V* is maintained between them. The plates are mutually parallel and co-axial, however their axis is perpendicular to the magnetic field. A photographic film, covers the inside of the curved surface of a cylinder of radius ρ + *s*, which is held co-axial with the plates. In other words, the film is at a radial distance *s* from the edges of the plates. The entire arrangement is placed *in vacuo*. Note that *t* is very much smaller than both *s* and *ρ*.

A point source of β particles, which emits the β particles uniformly in all directions with a range of velocities, is placed between the centres of the plates, and the *same piece of film* is exposed under three different conditions:

- firstly with $B = 0$, and $V = 0$,
- secondly with $B = B_0$, and $V = V_0$, and
- thirdly with $B = -B_0$, and $V = -V_0$;

where V_0 and B_0 are positive constants. Please note that the upper plate is positively charged when $V>0$ (negative when $V<0$), and that the magnetic field is in the direction defined by figure 2.5 when $B>0$ (in the opposite direction when $B<0$). For this part you may assume the gap is negligibly small.

Two regions of the film are labelled A and B on figure 2.3. After exposure and development, a sketch of one of these regions is given in figure 2.4. From which region was this piece taken (on your answer sheet write A or B)? Justify your answer by showing the directions of the forces acting on the electron.

c. After exposure and development, a sketch of the film is given in figure 2.4. Measurements are made of the separation of the two outermost traces with a microscope, and this distance (*y*) is also indicated for one particular angle on figure 2.4. The results are given in the table below, the angle ϕ being defined in figure 2.3 as the angle between the magnetic field and a line joining the centre of the plates to the point on the film.

Angle to field /degrees	ϕ	90	60	50	40	30	23
Separation /mm	*y*	17.4	12.7	9.7	6.4	3.3	End of trace

Numerical values of the system parameters are given below:

$B_0 = 6.91$ mT $V_0 = 580$ V $t = 0.80$ mm $s = 41.0$ mm

In addition, you may take the speed of light in vacuum to be 3.00×10^8 m s^{-1}, and the rest mass of the electron to be 9.11×10^{-31} kg.

Determine the maximum β particle kinetic energy observed.

Write the maximum kinetic energy as a numerical result in eV in the box on the answer sheet, section 2c.

d. Using the information given in part (c), obtain a value for the charge to rest mass ratio of the electron. This should be done by plotting an appropriate graph on the paper provided.

Indicate *algebraically* the quantities being plotted on the horizontal and vertical axes both on the graph itself *and* on the answer sheet in the boxes provided in section 2d.

Write your value for the charge to mass ratio of the electron in the box provided on the answer sheet, section 2d.

Please note that the answer you obtain may not agree with the accepted value because of a systematic error in the observations.

Additional Figures

Figure 2.3

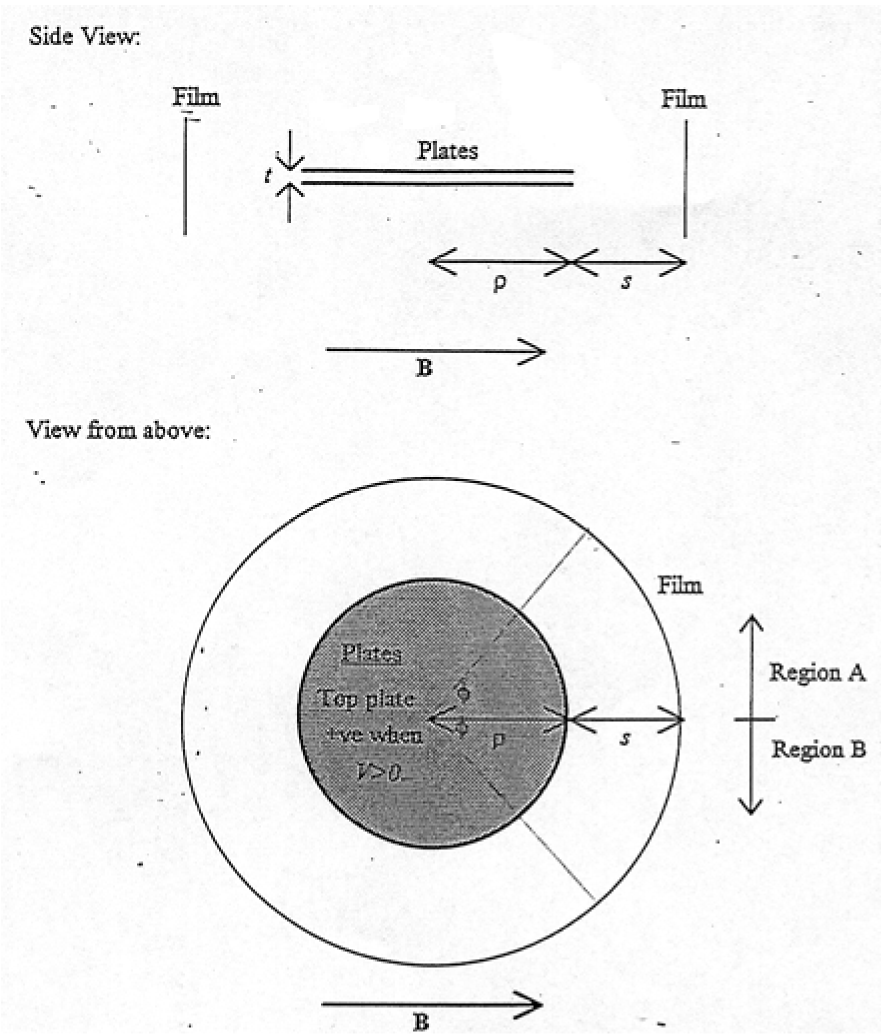
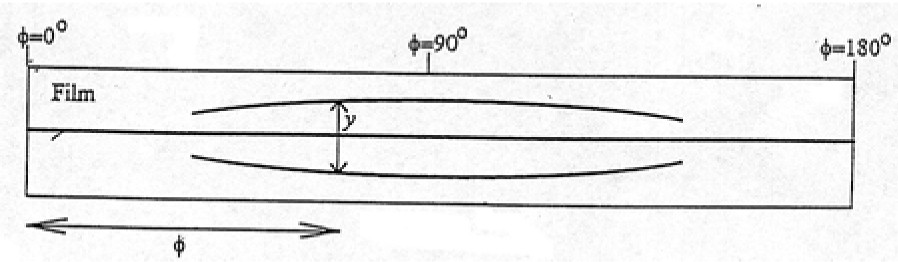


Figure 2.4


ϕ=0°
ϕ=90°
ϕ=180°
Film
y
ϕ

Theoretical Problem 3

Part A

This part is concerned with the difficulties of detecting gravitational waves generated by astronomical events. It should be realised that the explosion of a distant supernova may produce fluctuations in the gravitational field strength at the surface of the Earth of about 10^{-19} N kg^{-1}.A model for a gravitational wave detector (see figure 3.1) consists of two metal rods each 1m long, held at right angles to each other. One end of each rod is polished optically flat and the other end is held rigidly. The position of one rod is adjusted so there is a minimum signal received from the photocell (see figure 3.1).

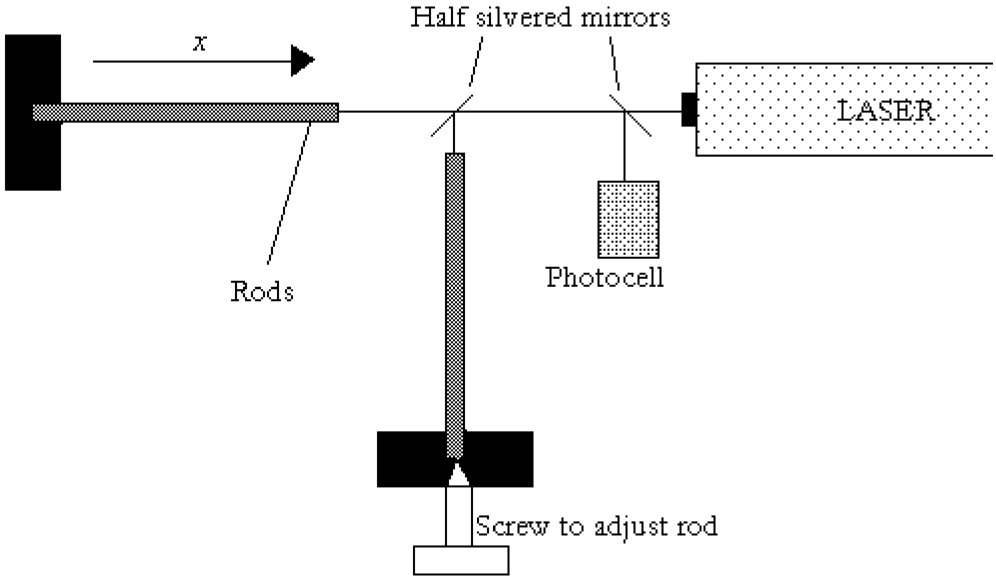


Figure 3.1

Figure 3.1

The rods are given a short sharp longitudinal impulse by a piezoelectric device. As a result the free ends of the rods oscillate with a longitudinal displacement Dx_t, where

$$\Delta x_t = a e^{-\mu t} \cos(\omega t + \phi),$$

and *a*, *m*, *w* and *f* are constants.

(a) If the amplitude of the motion is reduced by 20% during a 50s interval determine a value for *m*.

(b) Given that longitudinal wave velocity,v = Ö(E/ρ), determine also the lowest value for *w*, given that the rods are made of aluminium with a density (*r*) of 2700 kg.m^{-3} and a Young modulus (*E*) of 7.1 x 10^{10} Pa.

(c) It is impossible to make the rods exactly the same length so the photocell signal has a beat frequency of 0.005 Hz. What is the difference in length of the rods?

(d) For the rod of length *l*, derive an algebraic expression for the change in length, D*l*, due to a change, D*g*, in the gravitational field strength, *g*, in terms of *l* and other constants of the rod material. The response of the detector to this change takes place in the direction of one of the rods.

(e) The light produced by the laser is monochromatic with a wavelength of 656nm. If the minimum fringe shift that can be detected is 10^{-4} of the wavelength of the laser, what is the minimum value of *l* necessary if such a system were to be capable of detecting variations in *g* of 10^{-19} N kg^{-1}?

Part B

This part is concerned with the effect of a gravitational field on the propagation of light in space.

(a) A photon emitted from the surface of the Sun (mass M, radius R) is red-shifted. By assuming a rest-mass equivalent for the photon energy, apply Newtonian gravitational theory to show that the effective (or measured) frequency of the photon at infinity is reduced (red-shifted) by the factor $(1 - GM/Rc^2)$.

(b) A reduction of the photon's frequency is equivalent to an increase in its time period, or, using the photon as a standard clock, a dilation of time. In addition, it may be shown that a time dilation is always accompanied by a contraction in the unit of length by the same factor.

We will now try to study the effect that this has on the propagation of light near the Sun. Let us first define an effective refractive index n_r at a point r from the centre of the Sun. Let

$$n_r = \frac{c}{c_r'},$$

where c is the speed of light as measured by a co-ordinate system far away from the Sun's gravitational influence ($r \to \infty$), and c_r' is the speed of light as measured by a co-ordinate system at a distance r from the centre of the Sun.

Show that n_r may be approximated to:

$$n_r = 1 + \frac{\alpha GM}{rc^2},$$

for small GM/rc^2, where a is a constant that you determine.

(c) Using this expression for n_r, calculate in radians the deflection of a light ray from its straight path as it passes the edge of the Sun.

Data:

Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Mass of Sun, $M = 1.99 \times 10^{30}$ kg.

Radius of Sun, $R = 6.95 \times 10^{8}$ m.

Velocity of light, $c = 3.00 \times 10^{8}$ m s^{-1}.

You may also need the following integral

$$: \int_{-\infty}^{+\infty} \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{2}{a^2}.$$