I. Determination of e/k_B Through Electrolysis Process

Background Theory

The electrolysis of water is described by the reaction :

$$H_2O \rightarrow 2H^+ + O^{-2}$$

$$2H^+ + 2e^- \rightarrow H_2\,;\; O^{-2} \rightarrow \frac{1}{2}O_2 + 2e^-$$

The reaction takes place when an electric current is supplied through a pair of electrodes immersed in the water. Assume that both gases produced in the reaction are ideal.

One of the gases produced by the reaction is kept in a test tube marked by arbitrary scale. By knowing the total charge transferred and the volume of the gas in the test tube the quantity e/k_B can be determined, where **e** is the charge of electron and k_B is the Boltzmann constant.

For the purpose mentioned above, this experiment is divided into two parts.

Part A: Calibration of the arbitrary scale on the test tube by using a dynamic method.
This result will be used for part B

Part B: Determination of the physical quantity e/k_B by means of water electrolysis

You are not obliged to carry out the two experiments (part A and part B) in alphabetical order

The following physical quantities are assumed:

- Acceleration of gravity, g = (9.78 ± 0.01) ms^{-2}
- Ratio of internal vs external diameters of the test tube, $\alpha = 0.82 \pm 0.01$

The local values of temperature T and pressure P will be provided by the organizer.

List of tools and apparatus given for experiment (part A & B):

- Insulated copper wires of three different diameters:
  1. Brown of larger diameter
  2. Brown of smaller diameter
  3. Blue
- A regulated voltage source (0-60 V, max.1A)
- A plastic container and a bottle of water.
- A block of brass with plastic clamp to keep the electrode in place without damaging the insulation of the wire.
- A digital stopwatch.
- A multimeter (beware of its proper procedure).
- A wooden test tube holder designed to hold the tube vertically.
- A multipurpose pipette

- A vertical stand.
- A bottle of white correction fluid for marking.
- A cutter
- A pair of scissors
- A roll of cellotape
- A steel ball
- A piece of stainless steel plate to be used as electrode.
- A test tube with scales.
- Graph papers.

Note that all scales marked on the graph papers and the apparatus for the experiments (e.g. the test tube) are of the same scale unit, but ***not calibrated*** in millimeter.

EXPERIMENT

Part A: Calibration of the arbitrary scale on the test tube

- Determine a dynamic method capable of translating the arbitrary length scale to a known scale available.
- Write down an expression that relates the measurable quantities from your experiment in terms of the scale printed on the test tube, and sketch the experiment set up.
- Collect and analyze the data from your experiment for the determination and calibration of the unknown length scale.

Part B: Determination of physical quantity e/k_B

- Set up the electrolysis experiment with a proper arrangement of the test tube in order to trap one of the gases produced during the reaction.
- Derive an equation relating the quantities: time t, current I, and water level difference Dh, measured in the experiment.
- Collect and analyze the data from your experiment. For simplicity, you may assume that the gas pressure inside the tube remains constant throughout the experiment.
- Determine the value of e/k_B.

Country	Student No.	Experiment No.	Page No.	Total Pages

ANSWER FORM

PART A

1. State the method of your choice and sketch the experimental set up of the method: **[*0.5 pts*]**

2. Write down the expression relating the measurable quantities in your chosen method: **[*0.5 pts*]. State all the approximations used in obtaining this expression [1.0 pts].**

3. Collect and organize the data in the following orders : physical quantities, values, units **[*1.0 pts*]**

4. Indicate the quality of the calibration by showing the plot relating two independently measured quantities and mark the range of validity. **[*0.5 pts*]**

5. Determine the smallest unit of the arbitrary scale in term of mm and its estimated error induced in the measurements. **[*1.5 pts*]**

Country	Student No.	Experiment No.	Page No.	Total Pages

PART B

1. Sketch of the experimental set up. **[*1.0 pts*]**

2. Derive the following expression:

$$I\,\Delta t \;= \frac{\mathrm{e}}{\mathrm{k_B}}\,\frac{2P(pr^2)}{T}\,\Delta \mathrm{h}$$ **[*1.5 pts*]**

3. Collect and organize the data in the following format : physical quantities (value, units) **[*1.0 pts*]**

4. Determine the value of ***e/k_B*** and its estimated error **[*1.5 pts*]**

II. OPTICAL BLACK BOX

Description

In this problem, the students have to identify the unknown optical components inside the cubic box. The box is sealed and has only two narrow openings protected by red plastic covering. The components should be identified by means of optical phenomena observed in the experiment. Ignore the small thickness effect of the plastic covering layer.

A line going through the centers of the slits is defined as the axis of the box. Apart from the red plastic coverings, there are three (might be identical or different) elements from the following list:

- Mirror, either plane or spherical
- Lens, either positive or negative
- Transparent plate having parallel flat surfaces (so called plane-parallel plate)
- Prism
- Diffraction grating.

The transparent components are made of material with a refractive index of 1.47 at the wavelength used.

Apparatus available:

- A laser pointer with a wavelength of 670 nm. **CAUTION: DO NOT LOOK DIRECTLY INTO THE LASER BEAM.**
- An optical rail
- A platform for the cube, movable along the optical rail
- A screen which can be attached to the end of the rail, and detached from it for other measurements.
- A sheet of graph paper which can be pasted on the screen by cellotape.
- A vertical stand equipped with a universal clamp and a test tube with arbitrary scales, which are also used in the Problem I.

Note that all scales marked on the graph papers and the apparatus for the experiments are of the same scale unit, but ***not calibrated*** in millimeter.

The Problem

Identify each of the three components and give its respective specification:

Possible type of component	Specification required
mirror	radius of curvature, angle between the mirror axis and the axis of the box
lens*	positive or negative, its focal length, and its position inside the box
plane-parallel plate	thickness, the angle between the plate and the axis of the box
prism	apex angle, the angle between one of its deflecting sides and the axis of the box
diffraction grating*	line spacing, direction of the lines, and its position inside the box

- implies that its plane is at right angle to the axis of the box

Express your final answers for the specification parameters of each component (e.g. focal length, radius of curvature) in terms of millimeter, micrometer or the scale of graph paper.

You don't have to determine the accuracy of the results.

Country	Student No.	Experiment No.	Page No.	Total Pages

ANSWER FORM

1. Write down the types of the optical components inside the box :

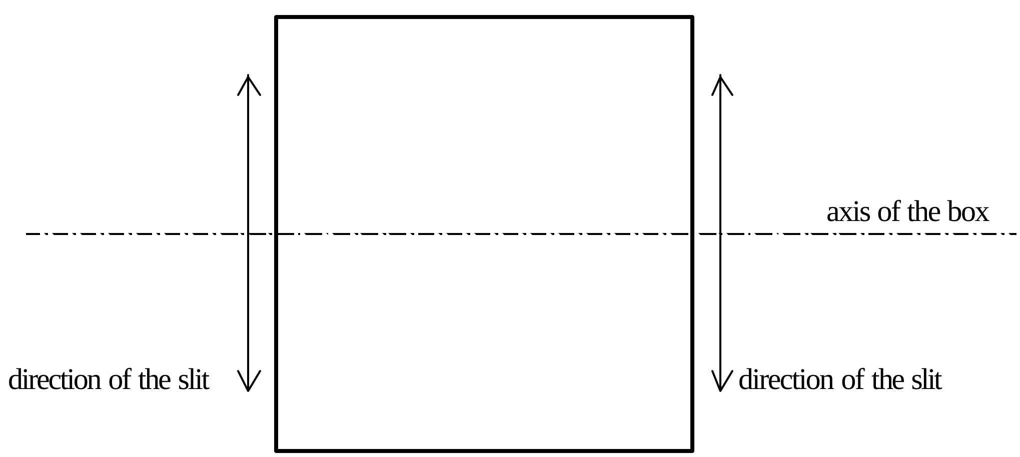
   no.1. .... **[*0.5 pts*]**

   no.2.. .... **[*0.5 pts*]**

   no.3. .... **[*0.5 pts*]**

2. The cross section of the box is given in the figure below. Add a sketch in the figure to show how the three components are positioned inside the box. In your sketch, denote each component with its code number in answer 1.

   [*0.5 pts* for each correct position]



Country	Student No.	Experiment No.	Page No.	Total Pages

3. Add detailed information with additional sketches regarding arrangement of the optical components in answer 2, such as the angle, the distance of the component from the slit, and the orientation or direction of the components. **[*1.0 pts*]**

Country	Student No.	Experiment No.	Page No.	Total Pages

4. Summarize the observed data **[*0.5 pts*]**, determine the specification of the optical component no.1 by deriving the appropriate formula with the help of drawing **[*1.0 pts*],** calculate the specifications in question and enter your answer in the box below **[*0.5 pts*]**.

Name of component no.1	Specification

Country	Student No.	Experiment No.	Page No.	Total Pages

5. Summarize the observed data **[*0.5 pts*]**, determine the specification of the optical component no.2 by deriving the appropriate formula with the help of drawing **[*1.0 pts*],** calculate the specifications in question and enter your answer in the box below **[*0.5 pts*]**.

Name of component no.2	Specification

Country	Student No.	Experiment No.	Page No.	Total Pages

6. Summarize the observed data **[*0.5 pts*]**, determine the specification of the optical component no.3 by deriving the appropriate formula with the help of drawing **[*1.0 pts*],** calculate the specifications in question and enter your answer in the box below **[*0.5 pts*]**.

Name of component no.3	Specification

I. Ground-Penetrating Radar

Ground-penetrating radar (GPR) is used to detect and locate underground objects near the surface by means of transmitting electromagnetic waves into the ground and receiving the waves reflected from those objects. The antenna and the detector are directly on the ground and they are located at the same point.

A linearly polarized electromagnetic plane wave of angular frequency ω propagating in the z direction is represented by the following expression for its field:

$$E = E_0\, e^{-a\,z} cos(wt - b\,z)\ , \qquad (1)$$

where E_o is constant, a is the attenuation coefficient and b is the wave number expressed respectively as follows

$$a = w\left\{\frac{me}{2}\left[\left(1+\frac{s^2}{e^2w^2}\right)^{1/2} - 1\right]\right\}^{1/2}\ ,\quad b = w\left\{\frac{me}{2}\left[\left(1+\frac{s^2}{e^2w^2}\right)^{1/2} + 1\right]\right\}^{1/2} \qquad (2)$$

with m, e, and s denoting the magnetic permeability, the electrical permittivity, and the electrical conductivity respectively.

The signal becomes undetected when the amplitude of the radar signal arriving at the object drops below 1/e (≈ 37%) of its initial value. An electromagnetic wave of variable frequency (10 MHz – 1000 MHz) is usually used to allow adjustment of range and resolution of detection.

The performance of GPR depends on its resolution. The resolution is given by the minimum separation between the two adjacent reflectors to be detected. The minimum separation should give rise to a minimum phase difference of 180^o between the two reflected waves at the detector.

Questions:

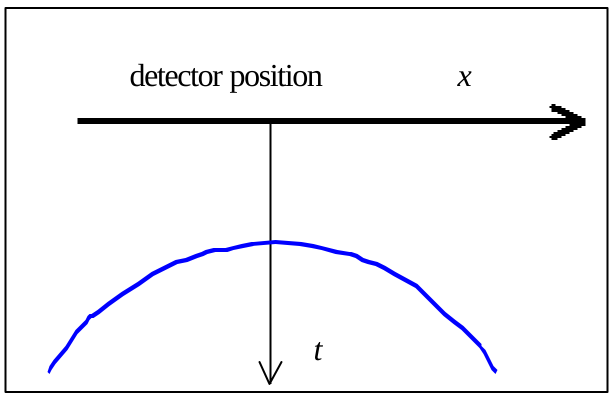
(Given : $m_0 = 4p \text{ x}10^{-7}$ H/m and $e_0 = 8.85 \text{ x}10^{-12}$ F/m)

1. Assume that the ground is non-magnetic ($m = m_0$) satisfying the condition $\left(\frac{s}{we}\right)^2 \langle\langle 1$. Derive the expression of propagation speed v in terms of m and e, using equations (1) and (2) **[*1.0 pts*]**.

2. Determine the maximum depth of detection of an object in the ground with conductivity of 1.0 mS/m and permittivity of $9\varepsilon_0$, satisfying the condition $\left(\frac{s}{we}\right)^2 \langle\langle 1$, (S=ohm^{-1} ; use $\mu=\mu_0$). **[*2.0 pts*]**

3. Consider two parallel conducting rods buried horizontally in the ground. The rods are 4 meter deep. The ground is known to have conductivity of 1.0 mS/m and permittivity of $9\varepsilon_0$. Suppose the GPR measurement is carried out at a position aproximately above one of the rod. Assume point detector is used. Determine the minimum frequency required to get a lateral resolution of 50 cm **[*3.5 pts*]**.

4. To determine the depth of a buried rod *d* in the same ground, consider the measurements carried out along a line perpendicular to the rod. The result is described by the following figure:



Graph of traveltime t vs detector position x, t_{min} = 100 ns.

Derive t as a function of x and determine d **[*3.5 pts*]**.

II. Sensing Electrical Signals

Some seawater animals have the ability to detect other creatures at some distance away due to electric currents produced by the creatures during the breathing processes or other processes involving muscular contraction. Some predators use this electrical signal to locate their preys, even when buried under the sands.

The physical mechanism underlying the current generation at the prey and its detection by the predator can be modeled as described by Figure II-1. The current generated by the prey flows between two spheres with positive and negative potential in the prey's body. The distance between the centers of the two spheres is l_s, each having a radius of r_s, which is much smaller than l_s. The seawater resistivity is r. Assume that the resistivity of the prey's body is the same as that of the surrounding seawater, implying that the boundary surrounding the prey in the figure can be ignored.

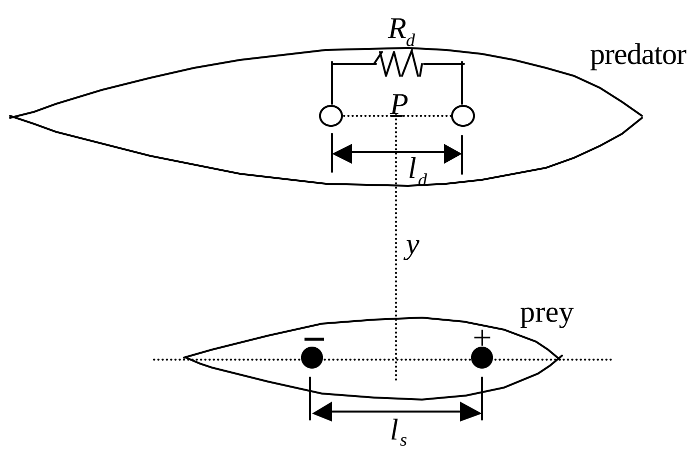


Figure II-1. A model describing the detection of electric power coming from a prey by its predator.

In order to describe the detection of electric power by the predator coming from the prey, the detector is modeled similarly by two spheres on the predator's body and in contact with the surrounding seawater, lying parallel to the pair in the prey's body. They are separated by a distance of l_d, each having a radius of r_d which is much smaller than l_d. In this case, the center of the detector is located at a distance y right above the source and the line connecting the two spheres is parallel to the electric field as shown in Figure II-1. Both l_s and l_d are also much smaller than y. The electric field strength along the line connecting the two spheres is assumed to be constant. Therefore the detector forms a closed circuit system connecting the prey, the surrounding seawater and the predator as described in Figure II-2.

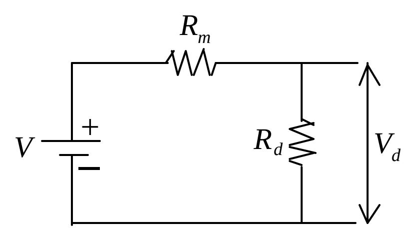


Figure II-2. The equivalent closed circuit system involving the sensing predator, the prey and the surrounding seawater.

In the figure, V is the voltage difference between the detector's spheres due to the electric field induced by the prey, R_m is the inner resistance due to the surrounding sea water. Further, V_d and R_d are respectively the voltage difference between the detecting spheres and the resistance of the detecting element within the predator.

Questions:

1. Determine the current density vector $\vec{j}$ (current per unit area) caused by a point current source I_s at a distance r in an infinite medium. **[*1.5 pts*]**

2. Based on the law $\vec{E} = r\vec{j}$, determine the electric field strength $\vec{E}_p$ at the middle of the detecting spheres (at point P) for a given current I_s that flows between two spheres in the prey's body **[*2.0 pts*]**.

3. Determine for the same current I_s, the voltage difference between the source spheres (V_s) in the prey **[*1.5 pts*]**. Determine the resistance between the two source spheres (R_s) **[*0.5 pts*]** and the power produced by the source (P_s) **[*0.5 pts*]**.

4. Determine R_m **[*0.5 pts*], ** V_d **[*1.0 pts*]** in Figure II-2 and calculate also the power transferred from the source to the detector (P_d) **[*0.5 pts*]**.

5. Determine the optimum value of R_d leading to maximum detected power **[*1.5 pts*]** and determine also the maximum power **[*0.5 pts*]**.

III. A Heavy Vehicle Moving on An Inclined Road

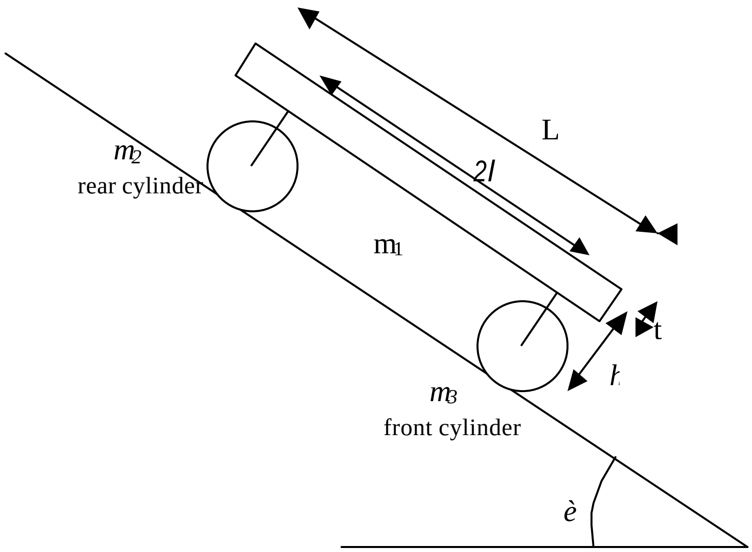


Figure III-1: A simplified model of a heavy vehicle moving on an inclined road.

The above figure is a simplified model of a heavy vehicle (road roller) with one rear and one front cylinder as its wheels on an inclined road with inclination angle of *è* as shown in Figure III-1. Each of the two cylinders has a total mass M(m_2=m_3=M) and consists of a cylindrical shell of outer radius R_o , inner radius R_i = 0.8 R_o and eight number of spokes with total mass 0.2 M. The mass of the undercarriage supporting the vehicle's body is negligible. The cylinder can be modeled as shown in Figure III-2. The vehicle is moving down the road under the influence of gravitational and frictional forces. The front and rear cylinder are positioned symmetrically with respect to the vehicle.

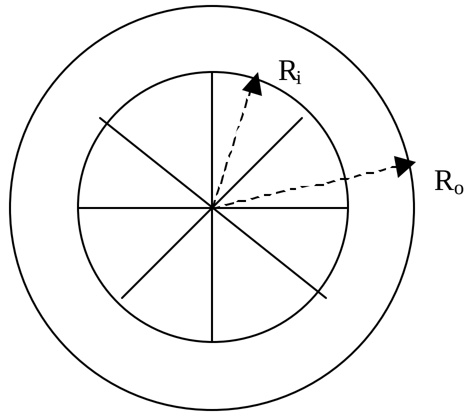


Figure III-2: A simplified model of the cylinders.

The static and kinetic friction coefficients between the cylinder *and* the road are m_s and m_k respectively. The body of the vehicle has a mass of 5M , length of L and thickness of t . The distance between the front and the rear cylinder is $2l$ while the distance from the center of cylinder to the base of the vehicle's body is h. Assume that the rolling friction between the cylinder and its axis is negligible.

Questions:

1. Calculate the moment of inertia of either cylinder **[*1.5 pts*]**.

2. Draw all forces that act on the body, the front cylinder, and the rear one. Write down equations of motion for each part of them **[*2.5 pts*]**.

3. The vehicle is assummed to move from rest, then freely move under gravitational influence. State all the possible types of motion of the system and derive their accelerations in terms of the given physical quantities **[*4.0 pts*]**.

4. Assume that after the vehicle travels a distance d by pure rolling from rest the vehicle enters a section of the road with all the friction coefficients drop to smaller constant values m_s' and m_k' such that the two cylinders start to slide. Calculate the linear and angular velocities of each cylinder after the vehicle has traveled a total distance of s meters. Here we assume that d and s is much larger than the dimension of vehicle **[*2.0 pts*]**