

34th International Physics Olympiad

TAIPEI, TAIWAN

Experimental Competition

Wednesday, August 6, 2003

Time Available : 5 hours

Read This First:

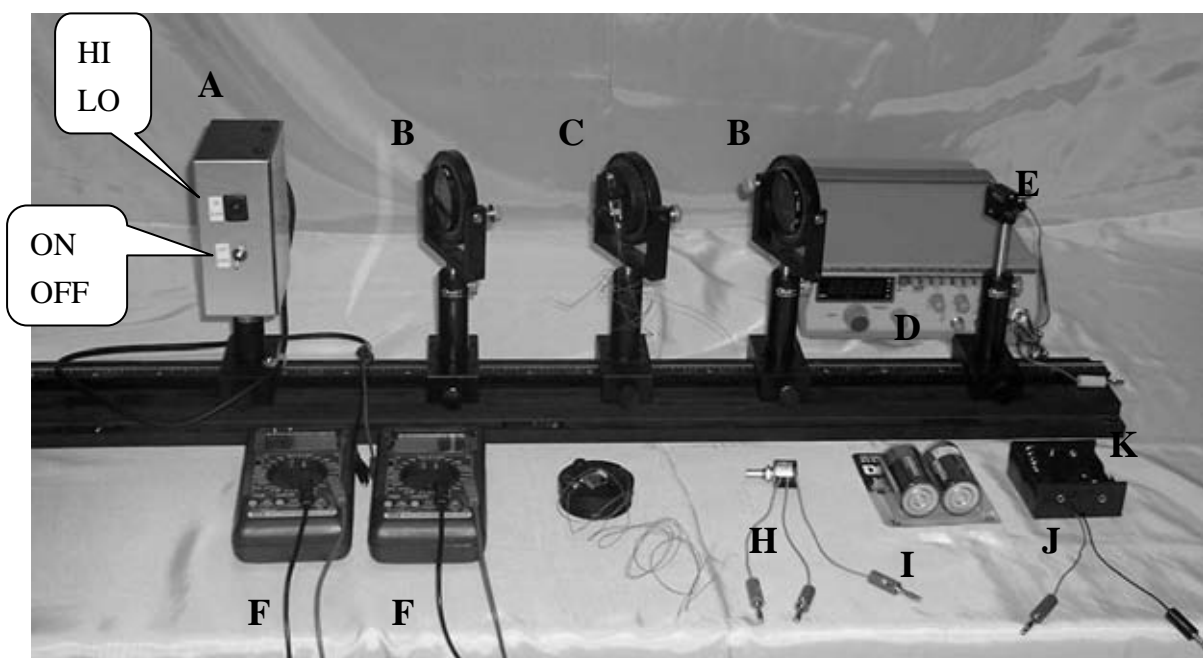
1. Use only the pen provided.
2. Use only the front side of the answer sheets and paper.
3. In your answers please use as little text as possible; express yourself primarily in equations, numbers and figures. If the required result is a numerical number, underline your final result with a wavy line.
4. Write on the blank sheets of paper the results of your measurements and whatever else you consider is required for the solution of the question and that you wish to be marked.
5. It is absolutely essential that you enter in the boxes at the top of each sheet of paper used your *Country* and your student number [*Student No.*]. In addition, on the blank sheets of paper used for each question, you should enter the question number [**Question No. : e.g. A-(1)**], the progressive number of each sheet [*Page No.*] and the total number of blank sheets that you have used and wish to be marked for each question [*Total No. of pages*]. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large cross through the whole sheet and do not include them in your numbering.
6. At the end of the exam please put your answer sheets and graphs in order.
7. Error bars on graphs are only needed in part A of the experiment.
8. **Caution: Do not look directly into the laser beam. You can damage your eyes!!**

Apparatuses and materials

1. Available apparatuses and materials are listed in the following table:

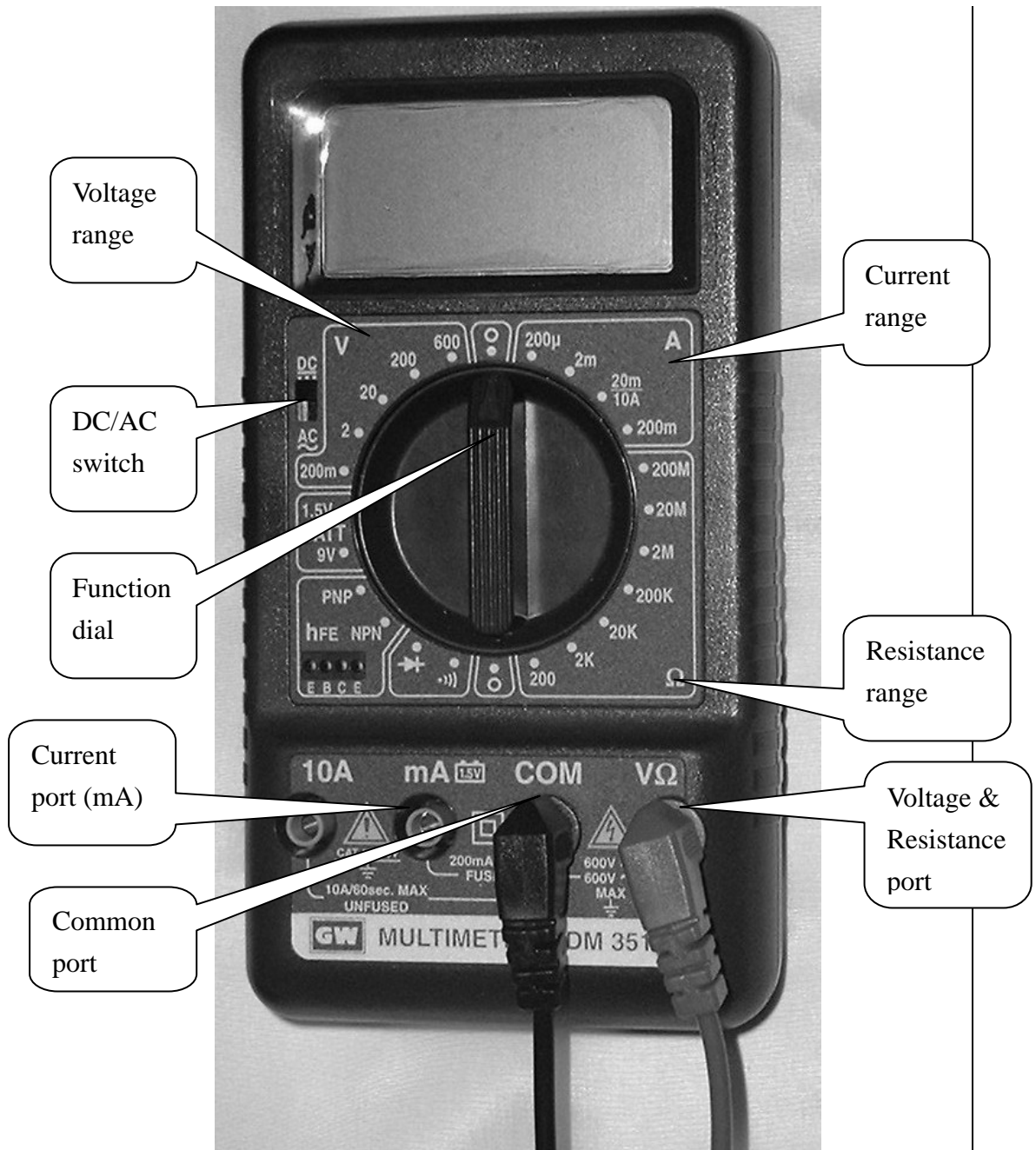
| | Apparatus & material | Quantity | | Apparatus & material | Quantity |
|---|--|----------|---|--|----------|
| A | Photodetector (PD) | 1 | I | Batteries | 2 |
| B | Polarizers with Rotary mount | 2 | J | Battery box | 1 |
| C | 90° TN-LC cell (yellow wires) with rotary LC mount | 1 | K | Optical bench | 1 |
| D | Function generator | 1 | L | Partially transparent papers | 2 |
| E | Laser diode (LD) | 1 | M | Ruler | 1 |
| F | Multimeters | 2 | N | White tape * (for marking on apparatus) | 1 |
| G | Parallel LC cell (orange wires) | 1 | O | Scissors | 1 |
| H | Variable resistor | 1 | P | Graph papers | 10 |

* Do not mark directly on apparatus. When needed, stick a piece of the white tape on the parts and mark on the white tape.



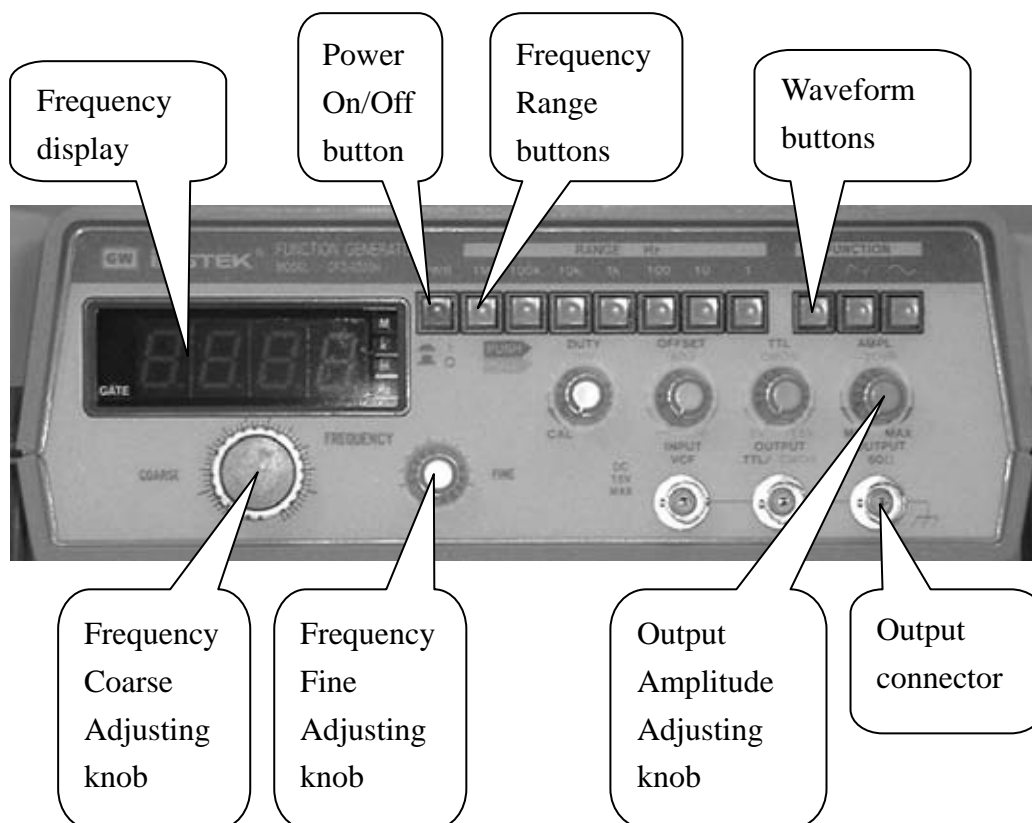
2. Instructions for the multimeter:

- “DC/AC” switch for selecting DC or AC measurement.
- Use the “V Ω ” and the “COM” inlets for voltage and resistance measurements.
- Use the “mA” and the “COM” inlets for small current measurements. The display then shows the current in milliamperes.
- Use the function dial to select the proper function and measuring range. “V” is for voltage measurement, “A” is for current measurement and “ Ω ” is for resistance measurement.



3. Instructions for the Function Generator:

- The power button may be pressed for “ON” and pressed again for “OFF”
- Select the frequencies range, and press the proper button.
- The frequency is shown on the digital display.
- Use the coarse and the fine frequency adjusting knobs to tune the proper frequency.
- Select the square-wave form by pressing the left most waveform button.
- Use the amplitude-adjusting knob to vary the output voltage.



Part A: Optical Properties of Laser Diode

I. Introduction

1. Laser Diode

The light source in this experiment is a laser diode which emits laser light with wavelength 650 nm. When the current of the laser diode (LD) is greater than the threshold current, the laser diode can emit monochromatic, partially polarized and coherent light. When the current in the laser diode is less than the threshold, the emitted light intensity is very small. At above the threshold current, the light intensity increases dramatically with the current and keeps a linear relationship with the current. If the current increases further, then the increasing rate of the intensity with respect to the current becomes smaller because of the higher temperature of the laser diode. Therefore, the optimal operating current range for the laser diode is the region where the intensity is linear with the current. In general, the threshold current I_{th} is defined as the intersection point of the current axis with the extrapolation line of the linear region.

Caution: Do not look directly into the laser beam. You can damage your eyes!!

2. Photodetector

The photodetector used in this experiment consists of a photodiode and a current amplifier. When an external bias voltage is applied on the photodiode, the photocurrent is generated by the light incident upon the diode. Under the condition of a constant temperature and monochromatic incident light, the photocurrent is proportional to the light intensity. On the other hand, the current amplifier is utilized to transfer the photocurrent into an output voltage. There are two transfer ratios in our photodetector – high and low gains. In our experiment, only the low gain is used. However, because of the limitation of the photodiode itself, the output voltage would go into saturation at about 8 Volts if the light intensity is too high and the photodiode cannot operate properly any more. Hence the appropriate operating range of the photodetector is when the output voltage is indeed proportional to the light intensity. If the light intensity is too high so that the photodiode reaches the saturation, the reading of the photodetector can not correctly represent the incident light intensity.

II. Experiments and procedures

Characteristics of the laser diode & the photodetector

In order to make sure the experiments are done successfully, the optical alignment of light rays between different parts of an experimental setup is crucial. Also the light source and the detector should be operated at proper condition. Part A is related to these questions and the question of the degree of polarization.

1. Mount the laser diode and photodetector in a horizontal line on the optical bench, as shown in Fig. 5. Connect the variable resistor, battery set, amperemeter, voltage meter, laser diode and photodetector according to Fig. 6. Adjust the variable resistor so that the current passing through LD is around 25 mA and the laser diode emits laser light properly. Choose the low gain for the photodetector. Align the laser diode and the photodetector to make the laser light level at the small hole on the detector box and the reading of the photodetector reaches a maximum value.

Caution: Do not let the black and the red leads of the battery contact with each other to avoid short circuit.

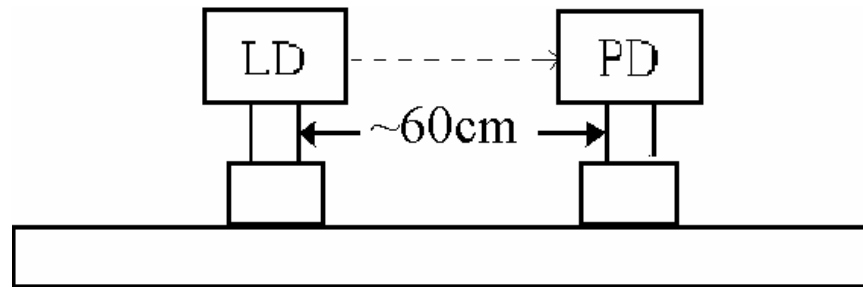


Fig. 5 Optical setup (LD : laser diode; PD : photodetector)

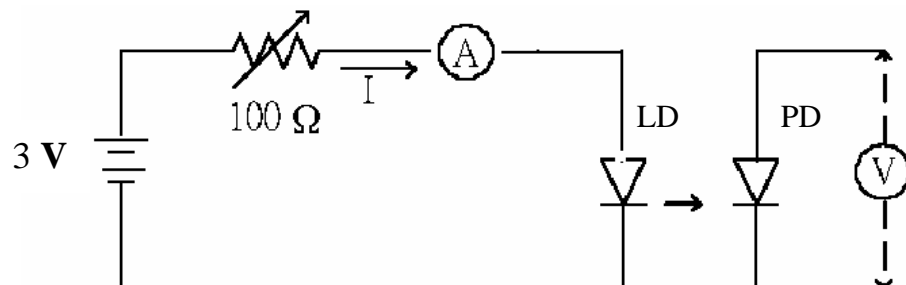


Fig. 6 Equivalent circuit for the connection of the laser diode

2. Use the output voltage of the photodetector to represent the laser light intensity J . Adjust the variable resistor to make the current I of the laser diode varying from

zero to a maximum value and measure the j as I increases. Be sure to choose appropriate current increment in the measurement.

Question A-(1) (1.5 point)

Measure, tabulate, and plot the j vs. I curve.

Question A-(2) (3.5 points)

Estimate the maximum current I_m with uncertainty in the linear region of the j vs. I curve. Mark the linear region on the $j - I$ curve figure by using arrows (\downarrow) and determine the threshold current I_{th} with uncertainty.

3. Choose the current of the laser diode as $I_{th} + 2(I_m - I_{th})/3$ to make sure the laser diode and photodetector are operated well.
4. **To prepare for the part B experiment:** Mount a polarizer on the optical bench close to the laser diode as shown in Fig. 7. Make sure the laser beam passing through the center portion of the polarizer. Adjust the polarizer so that the incident laser beam is perpendicular to the plane of the polarizer. (Hint: You can insert a piece of partially transparent paper as a test screen to check if the incident and reflected light spots coincide with each other.)

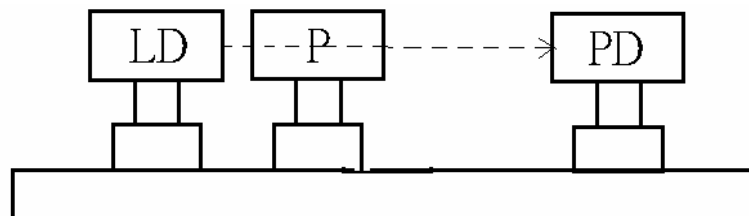


Fig. 7 Alignment of the polarizer (P : polarizer)

5. Keep the current of the laser diode unchanged, mount a second piece of polarizer on the optical bench and make sure proper alignment is accomplished, *i.e.*, set up the source, detector and polarizers in a straight line and make sure each polarizer plane is perpendicular to the light beam.

Part B Optical Properties of Nematic Liquid Crystal :

Electro-optical switching characteristic of 90° TN LC cell

I. Introduction

1. Liquid Crystal

Liquid crystal (LC) is a state of matter that is intermediate between the crystalline solid and the amorphous liquid. The nematic LCs are organic compounds consist of long-shaped needle-like molecules. The orientation of the molecules can be easily aligned and controlled by applying an electrical field. Uniform or well prescribed orientation of the LC molecules is required in most LC devices. The structure of the LC cell used in this experiment is shown in Fig 1. Rubbing the polyimide film can produce a well-aligned preferred orientation for LC molecules on substrate surfaces, thus due to the molecular interaction the whole slab of LC can achieve uniform molecular orientation. The local molecular orientation is called the director of LC at that point.

The LC cell exhibits the so-called double refraction phenomenon with two principal refractive indices. When light propagates along the direction of the director, all polarization components travel with the same speed $v_o = \frac{c}{n_o}$, where n_o is called the ordinary index of refraction. This propagation direction (direction of the director) is called the optic axis of the LC cell. When a light beam propagates in the direction perpendicular to the optic axis, in general, there are two speeds of propagation. The electric field of the light polarized perpendicular (or parallel) to the optic axis travels with the speed of $v_o = \frac{c}{n_o}$ (or $v_e = \frac{c}{n_e}$, where n_e is called the extraordinary index of refraction). The birefringence (optical anisotropy) is defined as the difference between the extraordinary and the ordinary indices of refraction $\Delta n \equiv n_e - n_o$.

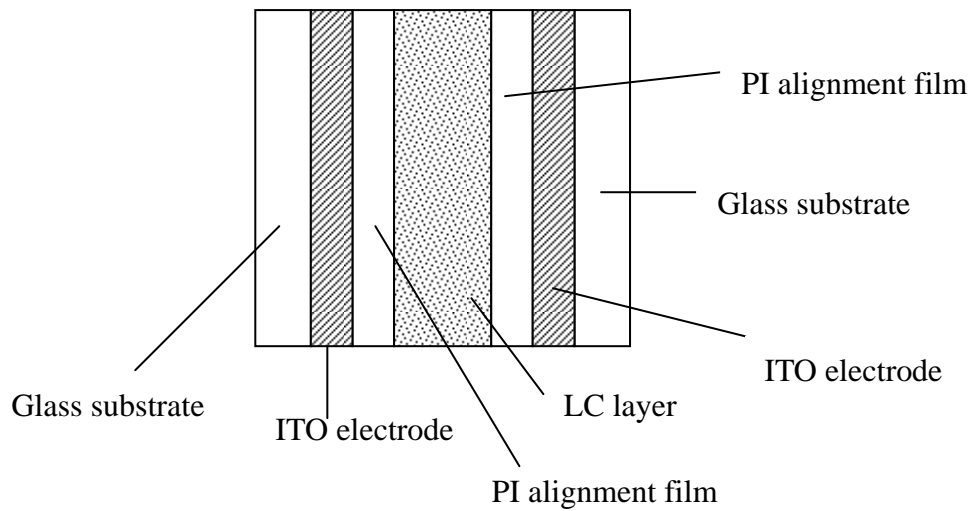


Fig. 1 LC cell structure

2. 90° Twisted Nematic LC Cell

In the 90° twisted nematic (TN) cell shown in Fig. 2, the LC director of the back surface is twisted 90° with respect to the front surface. The front local director is set parallel to the transmission axis of the polarizer. An incident unpolarized light is converted into a linearly polarized light by the front polarizer.

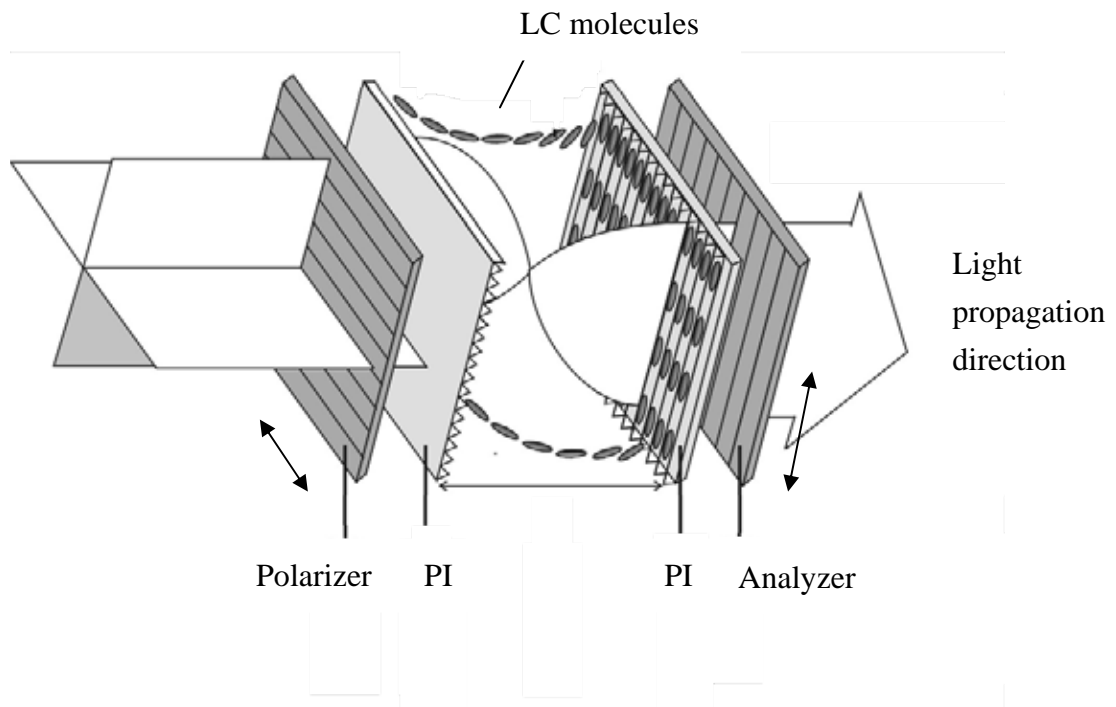


Fig. 2 90° TN LC cell

When a linearly polarized light traverses through a 90° TN cell, its polarization follows the twist of the LC directors (polarized light sees n_e only) so that the output beam remains linearly polarized except for that its polarization axis is rotated by 90° (it's called the polarizing rotary effect by n_e ; similarly we can also find polarizing rotary effect by n_o). Thus, for a normally black (NB) mode using a 90° TN cell, the analyzer's (a second polarizer) transmission axis is set to be parallel to the polarizer's transmission axis, as shown in Fig. 3 (see next page). However, when the applied voltage V across the LC cell exceeds a critical value V_c , the director of LC molecules tends to align along the direction of applied external electrical field which is in the direction of the propagation of light. Hence, the polarization guiding effect of the LC cell is gradually diminishing and the light leaks through the analyzer. Its electro-optical switching slope γ is defined as $(V_{90}-V_{10})/V_{10}$, where V_{10} and V_{90} are the applied voltages enabling output light signal reaches up to 10% and 90% of its maximum light intensity, respectively.

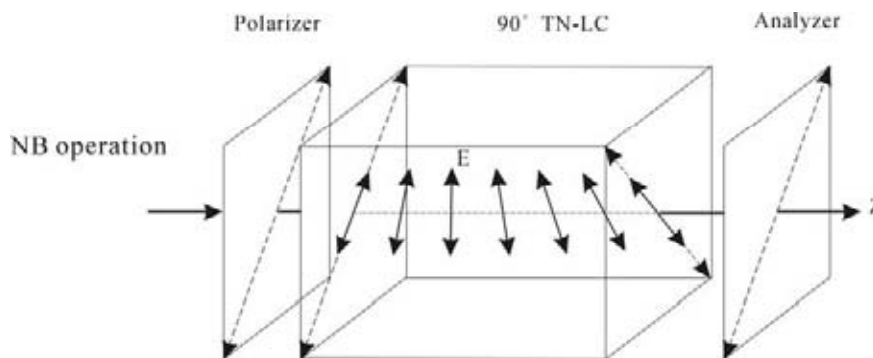


Fig. 3 NB mode operation of a 90° TN cell

II. Experiments and procedures

1. Setup a NB 90° TN LC mode between two polarizers with parallel transmission axes and apply 100 Hz square wave voltage using a function generator onto the ITO portions of two glass substrates and vary the applied voltage (V_{rms}) from 0 to 7.2 Volts.

* In the crucial turning points, take more data if necessary.

Question_B-(1) (5.0 points)

Measure, tabulate, and plot the electro-optical switching curve (J vs. V_{rms} curve) of the NB 90° TN LC, and find its switching slope γ , where γ is defined as $(V_{90} - V_{10})/V_{10}$.

Question B-(2) (2.5 points)

Determine the critical voltage V_c of this NB 90° TN LC cell. Show explicitly with graph how you determine the value V_c .

Hint:* When the external applied voltage exceeds the critical voltage, the light transmission increases rapidly and abruptly.

Part C Optical Properties of Nematic Liquid Crystal : **Electro-optical switching characteristic of parallel aligned LC cell**

I. Introduction

Homogeneous Parallel-aligned LC Cell

For a parallel-aligned LC cell, the directors in the front and back substrates are parallel with each other, as shown in Fig. 4. When a linearly polarized light impinges on a parallel-aligned cell with its polarization parallel to the LC director (rubbing direction), a pure phase modulation is achieved because the light behaves only as an extraordinary ray.

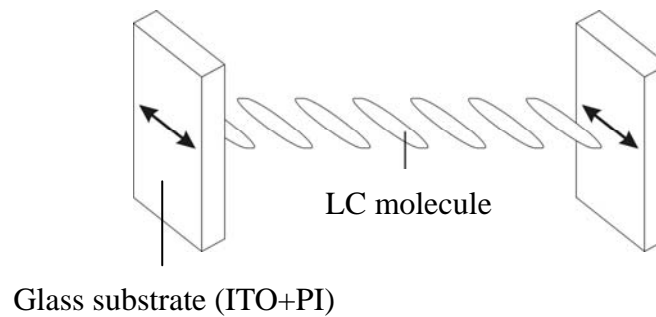


Fig. 4 Homogeneous parallel aligned LC cell

On the other hand, if a linearly polarized light is normally incident onto a parallel aligned cell but with its polarization making $\theta = 45^\circ$ relative to the direction of the aligned LC directors (refer to Fig. 8 in part C), then phase retardation occurs due to the different propagating speed of the extraordinary and ordinary rays in the LC medium. In this $\theta = 45^\circ$ configuration, when the two polarizers are parallel, the normalized transmission of a parallel aligned LC cell is given by

$$T_{\parallel} = \cos^2 \frac{\delta}{2}$$

The phase retardation δ is expressed as

$$\delta = 2\pi d \Delta n(V, \lambda) / \lambda$$

where d is the LC layer thickness, λ is the wavelength of light in air, V is the root mean square of applied AC voltage, and Δn , a function of λ and V , is the LC birefringence. It should be also noted that, at $V = 0$, $\Delta n (= n_e - n_o)$ has its maximum value, so does δ . Also Δn decreases as V increases.

In the general case, we have

$$T_{\parallel} = 1 - \sin^2 2\theta \sin^2 \frac{\delta}{2}$$

$$T_{\perp} = \sin^2 2\theta \sin^2 \frac{\delta}{2}$$

where \parallel and \perp represent that the transmission axis of analyzer is parallel and perpendicular to that of the polarizer, respectively.

II. Experiments and procedures

1. Replace NB 90° TN LC cell with parallel-aligned LC cell.
2. Set up $\theta = 45^\circ$ configuration at $V=0$ as shown in Fig. 8. Let the analyzer's transmission axis perpendicular to that of the polarizer, then rotate the parallel-aligned LC cell until the intensity of the transmitted light reaches the maximum value (T_{\perp}). This procedure establishes the $\theta = 45^\circ$ configuration. Take down T_{\perp} value, then, measure the intensity of the transmitted light (T_{\parallel}) of the same LC cell at the analyzer's transmission axis parallel to that of the polarizer (also at $V = 0$).

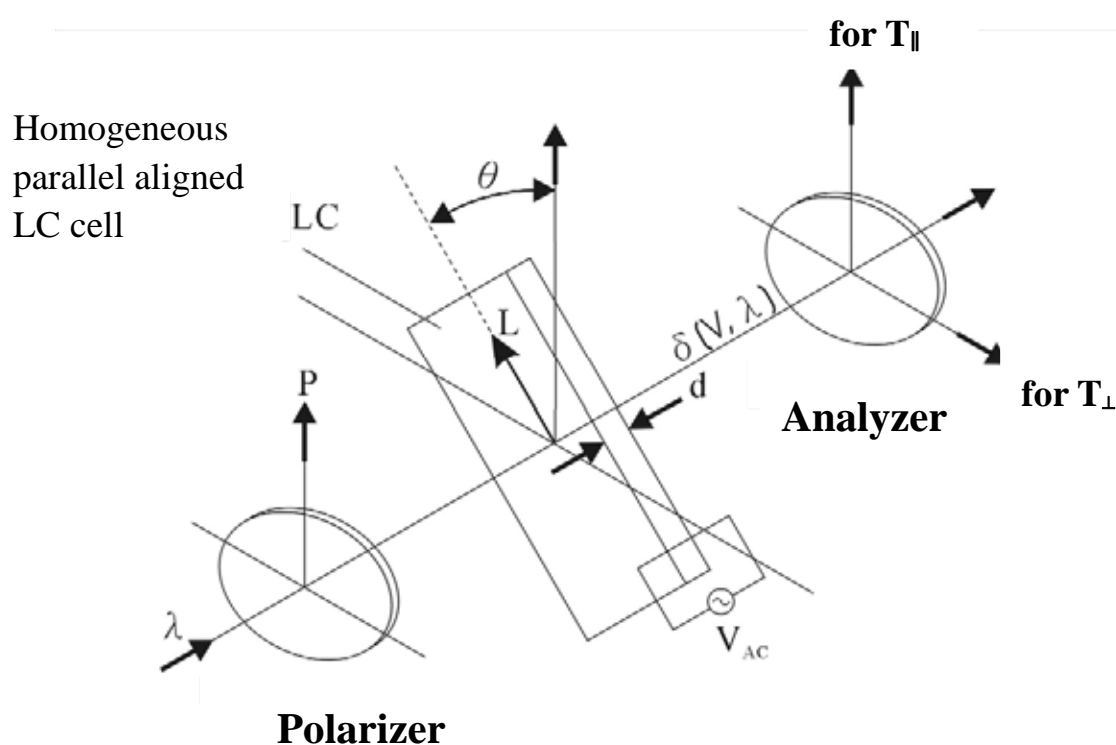


Fig. 8 Schematic diagram of experimental setup.

(The arrow L is the alignment direction)

Question C-(1) (2.5 points)

Assume that the wavelength of laser light 650 nm, LC layer thickness 7.7 μm , and approximate value of $\Delta n \approx 0.25$ are known. From the experimental data T_{\perp} and T_{\parallel} obtained above, calculate the accurate value of the phase retardation δ and accurate value of birefringence Δn of this LC cell at $V=0$.

3. Similar to the above experiment (1), in the $\theta = 45^{\circ}$ configuration, apply 100 Hz square wave voltage using a function generator onto the ITO portions of two glass substrates, vary the applied voltage (V_{rms}) from 0 to 7 Volts and measure the electro-optical switching curve (T_{\parallel}) at the analyzer's transmission axis parallel to the polarizer's transmission axis. (Hint: Measuring the T_{\perp} switching curve is helpful to increase the data accuracy of the above T_{\parallel} measurement; the data of T_{\perp} are not needed in the following questions.)

*** In the crucial turning points, take more data if necessary (especially in the range of 0.5-4.0 Volts).**

Question C-(2) (3.0 points)

Measure, tabulate, and plot the electro-optical switching curve for T_{\parallel} of this parallel aligned LC cell in the $\theta = 45^{\circ}$ configuration.

Question C-(3) (2.0 points)

From the electro-optical switching data, find the value of the external applied voltage V_{π} .

Hint: * V_{π} is the applied voltage which enables the phase retardation of this anisotropic LC cell become π (or 180°).

* Remember that Δn is a function of applied voltage, and Δn decreases as V increases.

* Interpolation is probably needed when you determine the accurate value of this V_{π} .

Theoretical Question 1
A Swing with a Falling Weight

A rigid cylindrical rod of radius R is held horizontal above the ground. With a string of negligible mass and length L ($L > 2\pi R$), a pendulum bob of mass m is suspended from point A at the top of the rod as shown in Figure 1a. The bob is raised until it is level with A and then released from rest when the string is taut. Neglect any stretching of the string. Assume the pendulum bob may be treated as a mass point and swings only in a plane perpendicular to the axis of the rod. Accordingly, the pendulum bob is also referred to as the *particle*. The acceleration of gravity is \vec{g} .

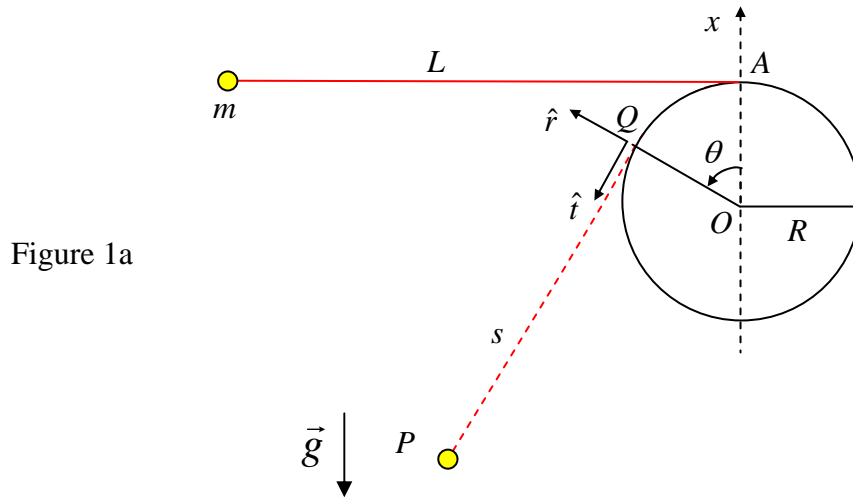


Figure 1a

Let O be the origin of the coordinate system. When the particle is at point P , the string is tangential to the cylindrical surface at Q . The length of the line segment QP is called s . The unit tangent vector and the unit radial vector at Q are given by \hat{t} and \hat{r} , respectively. The angular displacement θ of the radius OQ , as measured counterclockwise from the vertical x -axis along OA , is taken to be positive.

When $\theta = 0$, the length s is equal to L and the gravitational potential energy U of the particle is zero. As the particle moves, the instantaneous time rates of change of θ and s are given by $\dot{\theta}$ and \dot{s} , respectively.

Unless otherwise stated, all the speeds and velocities are relative to the fixed

point O .

Part A

In Part A, the string is taut as the particle moves. In terms of the quantities introduced above (i.e., s , θ , \dot{s} , $\dot{\theta}$, R , L , g , \hat{i} and \hat{r}), find:

- (a) The relation between $\dot{\theta}$ and \dot{s} . [0.5 point]
- (b) The velocity \vec{v}_Q of the moving point Q relative to O . [0.5 point]
- (c) The particle's velocity \vec{v}' relative to the moving point Q when it is at P . [0.7 point]
- (d) The particle's velocity \vec{v} relative to O when it is at P . [0.7 point]
- (e) The \hat{i} -component of the particle's *acceleration* relative to O when it is at P . [0.7 point]
- (f) The particle's gravitational potential energy U when it is at P . [0.5 point]
- (g) The speed v_m of the particle at the lowest point of its trajectory. [0.7 point]

Part B

In Part B, the ratio L to R has the following value:

$$\frac{L}{R} = \frac{9\pi}{8} + \frac{2}{3} \cot \frac{\pi}{16} = 3.534 + 3.352 = 6.886$$

- (h) What is the speed v_s of the particle when the string segment from Q to P is both straight and shortest in length? (in terms of g and R) [2.4 points]
- (i) What is the speed v_H of the particle at its highest point H when it has swung to the other side of the rod? (in terms of g and R) [1.9 points]

Part C

In Part C, instead of being suspended from A , the pendulum bob of mass m is connected by a string over the top of the rod to a heavier weight of mass M , as shown in Figure 1b. The weight can also be treated as a particle.

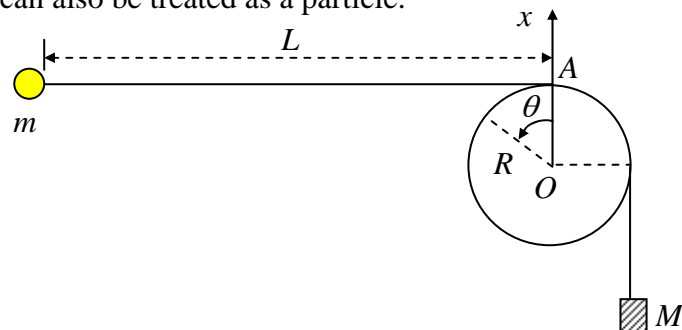


Figure 1b

Initially, the bob is held stationary at the same level as A so that, with the weight hanging below O , the string is taut with a horizontal section of length L . The bob is then released from rest and the weight starts falling. Assume that the bob remains in a vertical plane and can swing past the falling weight without **any interruption**.

The kinetic friction between the string and the rod surface is negligible. But the static friction is assumed to be large enough so that the weight will remain stationary once it has come to a stop (i.e. zero velocity).

- (j) Assume that the weight indeed comes to a stop after falling a distance D and that $(L-D) \gg R$. If the particle can then swing around the rod to $\theta = 2\pi$ while both segments of the string free from the rod remain straight, the ratio $\alpha = D/L$ must not be smaller than a critical value α_c . Neglecting terms of the order R/L or higher, obtain an estimate on α_c in terms of M/m . [3.4 points]

[Answer Sheet]

Theoretical Question 1
A Swing with a Falling Weight

(a) The relation between $\dot{\theta}$ and \dot{s} is

(b) The velocity of the moving point Q relative to O is

$$\vec{v}_Q =$$

(c) When at P , the particle's velocity relative to the moving point Q is

$$\vec{v}' =$$

(d) When at P , the particle's velocity relative to O is

$$\vec{v} =$$

(e) When at P , the \hat{t} -component of the particle's acceleration relative to O is

(f) When at P , the particle's gravitational potential energy is

$$U =$$

(g) The particle's speed when at the lowest point of its trajectory is

$$v_m =$$

(h) When line segment QP is straight with the shortest length, the particle's speed is
(Give expression and value in terms of g and R)

$$v_s =$$

(i) At the highest point, the particle's speed is (Give expression and value in terms of g and R)

$$v_H =$$

(j) In terms of the mass ratio M/m , the critical value α_c of the ratio D/L is

$$\alpha_c =$$

Theoretical Question 2

A Piezoelectric Crystal Resonator under an Alternating Voltage

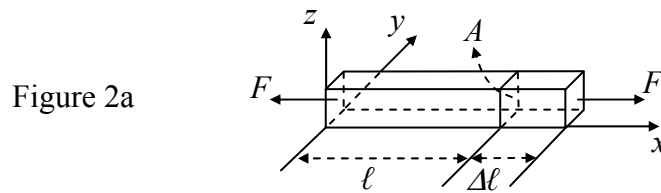
Consider a uniform rod of unstressed length ℓ and cross-sectional area A (Figure 2a). Its length changes by $\Delta\ell$ when equal and opposite forces of magnitude F are applied to its ends faces normally. The *stress* T on the end faces is defined to be F/A . The *fractional change* in its length, i.e., $\Delta\ell/\ell$, is called the *strain* S of the rod. In terms of stress and strain, Hooke's law may be expressed as

$$T = Y S \quad \text{or} \quad \frac{F}{A} = Y \frac{\Delta\ell}{\ell} \quad (1)$$

where Y is called the *Young's modulus* of the rod material. Note that a *compressive* stress T corresponds to $F < 0$ and a decrease in length (i.e., $\Delta\ell < 0$). Such a stress is thus negative in value and is related to the pressure p by $T = -p$.

For a uniform rod of density ρ , the speed of propagation of longitudinal waves (i.e., sound speed) along the rod is given by

$$u = \sqrt{Y / \rho} \quad (2)$$



The effect of damping and dissipation can be ignored in answering the following questions.

Part A: mechanical properties

A uniform rod of semi-infinite length, extending from $x = 0$ to ∞ (see Figure 2b), has a density ρ . It is initially stationary and unstressed. A piston then steadily exerts a small pressure p on its left face at $x = 0$ for a very short time Δt , causing a pressure wave to propagate with *speed* u to the right.

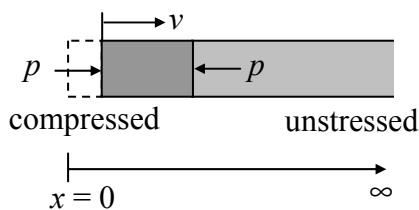


Figure 2b

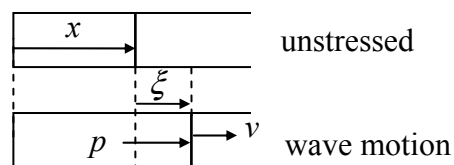


Figure 2c

- (a) If the piston causes the rod's left face to move at a constant velocity v (Figure 2b), what are the strain S and pressure p at the left face during the time Δt ?

Answers must be given in terms of ρ , u , and v only. [1.6 points]

- (b) Consider a longitudinal wave traveling along the x direction in the rod. For a cross section at x when the rod is unstressed (Figure 2c), let $\xi(x, t)$ be its

displacement at time t and assume

$$\xi(x, t) = \xi_0 \sin k(x - ut) \quad (3)$$

where ξ_0 and k are constants. Determine the corresponding velocity $v(x, t)$, strain $S(x, t)$, and pressure $p(x, t)$ as a function of x and t . [2.4 points]

Part B: electromechanical properties (including piezoelectric effect)

Consider a quartz crystal slab of length b , thickness h , and width w (Figure 2d). Its length and thickness are along the x -axis and z -axis. Electrodes are formed by thin metallic coatings at its top and bottom surfaces. Electrical leads that also serve as mounting support (Figure 2e) are soldered to the electrode's centers, which may be assumed to be stationary for longitudinal oscillations along the x direction.

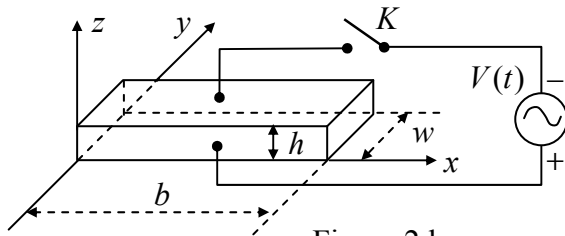


Figure 2d

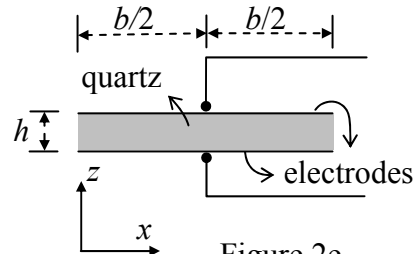


Figure 2e

The quartz crystal under consideration has a density ρ of $2.65 \times 10^3 \text{ kg/m}^3$ and Young's modulus Y of $7.87 \times 10^{10} \text{ N/m}^2$. The length b of the slab is 1.00 cm and the width w and height h of the slab are such that $h \ll w$ and $w \ll b$. With switch K left open, we assume only *longitudinal modes* of standing wave oscillation in the x direction are excited in the quartz slab.

For a standing wave of frequency $f = \omega/2\pi$, the displacement $\xi(x, t)$ at time t of a cross section of the slab with equilibrium position x may be written as

$$\xi(x, t) = 2\xi_0 g(x) \cos \omega t, \quad (0 \leq x \leq b) \quad (4a)$$

where ξ_0 is a positive constant and the spatial function $g(x)$ is of the form

$$g(x) = B_1 \sin k\left(x - \frac{b}{2}\right) + B_2 \cos k\left(x - \frac{b}{2}\right). \quad (4b)$$

$g(x)$ has the maximum value of one and $k = \omega/u$. Keep in mind that the centers of the electrodes are stationary and the left and right faces of the slab are free and must have zero stress (or pressure).

(c) Determine the values of B_1 and B_2 in Eq. (4b) for a longitudinal standing wave in the quartz slab. [1.2 point]

(d) What are the two lowest frequencies at which longitudinal standing waves may be excited in the quartz slab? [1.2 point]

The *piezoelectric* effect is a special property of a *quartz* crystal. Compression or dilatation of the crystal generates an electric voltage across the crystal, and conversely, an external voltage applied across the crystal causes the crystal to expand or contract depending on the polarity of the voltage. Therefore, mechanical and electrical oscillations can be coupled and made to resonate through a quartz crystal.

To account for the piezoelectric effect, let the surface charge densities on the upper and lower electrodes be $-\sigma$ and $+\sigma$, respectively, when the quartz slab is under an electric field E in the z direction. Denote the slab's strain and stress in the x direction by S and T , respectively. Then the piezoelectric effect of the quartz crystal can be described by the following set of equations:

$$S = (1/Y)T + d_p E \quad (5a)$$

$$\sigma = d_p T + \epsilon_T E \quad (5b)$$

where $1/Y = 1.27 \times 10^{-11} \text{ m}^2/\text{N}$ is the *elastic compliance* (i.e., inverse of Young's modulus) at constant electric field and $\epsilon_T = 4.06 \times 10^{-11} \text{ F/m}$ is the *permittivity* at constant stress, while $d_p = 2.25 \times 10^{-12} \text{ m/V}$ is the *piezoelectric coefficient*.

Let switch K in Fig. 2d be closed. The alternating voltage $V(t) = V_m \cos \omega t$ now acts across the electrodes and a *uniform* electric field $E(t) = V(t)/h$ in the z direction appears in the quartz slab. When a steady state is reached, a longitudinal standing wave oscillation of angular frequency ω appears in the slab in the x direction.

With E being uniform, the wavelength λ and the frequency f of a longitudinal standing wave in the slab are still related by $\lambda = u/f$ with u given by Eq. (2). But, as Eq. (5a) shows, $T = YS$ is no longer valid, although the definitions of strain and stress remain unchanged and the end faces of the slab remain free with zero stress.

(e) Taking Eqs. (5a) and (5b) into account, the surface charge density σ on the lower electrode as a function of x and t is of the form,

$$\sigma(x, t) = \left[D_1 \cos k \left(x - \frac{b}{2} \right) + D_2 \right] \frac{V(t)}{h},$$

where $k = \omega/u$. Find the expressions for D_1 and D_2 . [2.2 points]

(f) The total surface charge $Q(t)$ on the lower electrode is related to $V(t)$ by

$$Q(t) = \left[1 + \alpha^2 \left(\frac{2}{kb} \tan \frac{kb}{2} - 1 \right) \right] C_0 V(t) \quad (6)$$

Find the expression for C_0 and the expression and numerical value of α^2 .

[1.4 points]

A Piezoelectric Crystal Resonator under an Alternating Voltage

Wherever requested, give each answer as analytical expressions followed by numerical values and units. For example: area of a circle $A = \pi r^2 = 1.23 \text{ m}^2$.

- (a) The strain S and pressure p at the left face are (in terms of ρ , u , and v)

| |
|-------|
| $S =$ |
| $p =$ |

- (b) The velocity $v(x, t)$, strain $S(x, t)$, and pressure $p(x, t)$ are

| |
|-------------|
| $v(x, t) =$ |
| $S(x, t) =$ |
| $p(x, t) =$ |

- (c) The values of B_1 and B_2 are

| |
|---------|
| $B_1 =$ |
| $B_2 =$ |

- (d) The lowest two frequencies of standing waves are (expression and value)

| |
|-------------------|
| The Lowest |
| The Second Lowest |

- (e) The expressions of D_1 and D_2 are

| |
|---------|
| $D_1 =$ |
| $D_2 =$ |

- (f) The constants α^2 (expression and value) and C_0 are (expression only)

| |
|--------------|
| $\alpha^2 =$ |
| $C_0 =$ |

Theoretical Question 3

Part A

Neutrino Mass and Neutron Decay

A free neutron of mass m_n decays at rest in the laboratory frame of reference into three non-interacting particles: a proton, an electron, and an anti-neutrino. The rest mass of the proton is m_p , while the rest mass of the anti-neutrino m_ν is assumed to be nonzero and much smaller than the rest mass of the electron m_e . Denote the speed of light in vacuum by c . The measured values of mass are as follows:

$$m_n = 939.56563 \text{ MeV}/c^2, m_p = 938.27231 \text{ MeV}/c^2, m_e = 0.5109907 \text{ MeV}/c^2$$

In the following, all energies and velocities are referred to the laboratory frame. Let E be the total energy of the electron coming out of the decay.

- (a) Find the maximum possible value E_{\max} of E and the speed v_m of the anti-neutrino when $E = E_{\max}$. Both answers must be expressed in terms of the rest masses of the particles and the speed of light. Given that $m_\nu < 7.3 \text{ eV}/c^2$, compute E_{\max} and the ratio v_m/c to 3 significant digits. [4.0 points]

Part B

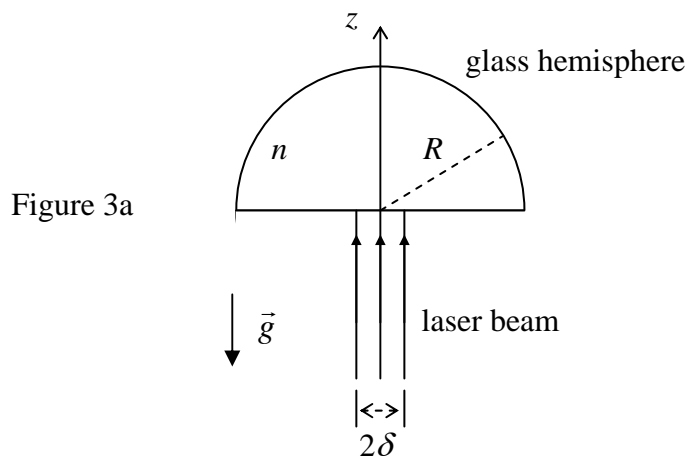
Light Levitation

A transparent glass hemisphere with radius R and mass m has an index of refraction n . In the medium outside the hemisphere, the index of refraction is equal to one. A parallel beam of monochromatic laser light is incident uniformly and normally onto the central portion of its planar surface, as shown in Figure 3a. The acceleration of gravity \vec{g} is vertically downwards. The radius δ of the circular cross-section of the laser beam is much smaller than R . Both the glass hemisphere and the laser beam are axially symmetric with respect to the z -axis.

The glass hemisphere does not absorb any laser light. Its surface has been coated with a thin layer of transparent material so that reflections are negligible when light enters and leaves the glass hemisphere. The optical path traversed by laser light passing through the non-reflecting surface layer is also negligible.

(b) Neglecting terms of the order $(\delta/R)^3$ or higher, find the laser power P needed to balance the weight of the glass hemisphere. [4.0 points]

Hint: $\cos \theta \approx 1 - \theta^2/2$ when θ is much smaller than one.



Wherever requested, give each answer as analytical expressions followed by numerical values and units. For example: area of a circle $A = \pi r^2 = 1.23 \text{ m}^2$.

Neutrino Mass and Neutron Decay

(a) (Give expressions in terms of rest masses of the particles and the speed of light)

The maximum energy of the electron is (*expression and value*)

| |
|--------------------|
| $E_{\text{max}} =$ |
|--------------------|

The ratio of anti-neutrino's speed at $E = E_{\text{max}}$ to c is (*expression and value*)

| |
|--------------------|
| $v_{\text{m}}/c =$ |
|--------------------|

Light Levitation

(b) The laser power needed to balance the weight of the glass hemisphere is

| |
|-------|
| $P =$ |
|-------|