35th International Physics Olympiad

Pohang, Korea

15 ~ 23 July 2004

Experimental Competition

Monday, 19 July 2004

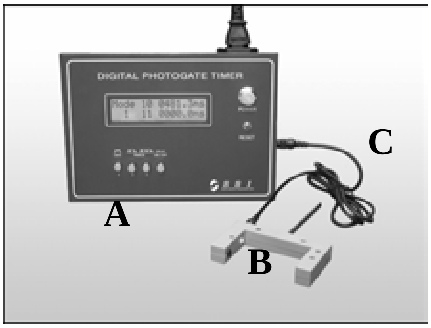
Please, first read the following instruction carefully:

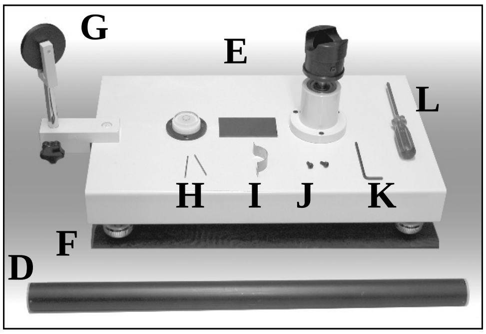
1. The time available is 5 hours.
2. Use only the pen provided.
3. Use only the front side of the ***writing sheets***. Write only inside the boxed area.
4. In addition to the ***blank writing sheets***, there are ***Answer Forms*** where you *must* summarize the results you have obtained.
5. Write on the ***blank writing sheets*** the results of your measurements and whatever else you consider is required for the solution to the question. Please, use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
6. In the boxes at the top of each sheet of paper write down your country code (***Country Code***) and student number (***Student Code***). In addition, on each blank ***writing sheets***, write down the progressive number of each sheet (***Page Number***) and the total number of ***writing sheets*** used (***Total Number of Pages***). If you use some blank ***writing sheets*** for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
7. At the end of the experiment, arrange all sheets *in the following order:*
   - *Answer forms* (top)
   - used *writing sheets* in order
   - the sheets you do not wish to be marked
   - unused *writing sheets*
   - the printed question (bottom)
8. It is not necessary to specify the error range of your values. However, their deviations from the actual values will determine your mark.
9. Place the papers inside the envelope and leave everything on your desk. **You are not allowed to take *any* sheet of paper or *any* material used in the experiment out of the room.**

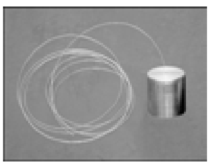
Apparatus and materials

1. List of available apparatus and materials

	Name	Quantity		Name	Quantity
A	Photogate timer	1	L	Philips screw driver	1
B	Photogate	1	M	Weight with a string	1
C	Connecting cable	1	N	Electronic balance	1
D	Mechanical "black box" (Black cylinder)	1	O	Stand with a ruler	1
E	Rotation stage	1	P	U-shaped support	1
F	Rubber pad	1	Q	C-clamp	1
G	Pulley	2		Ruler (0.50 m, 0.15 m)	1 each
H	Pin	2		Vernier calipers	1
I	U-shaped plate	1		Scissors	1
J	Screw	2		Thread	1
K	Allen (hexagonal, L-shaped) wrench	1		Spares (string, thread, pin, screw, Allen wrench)	







M

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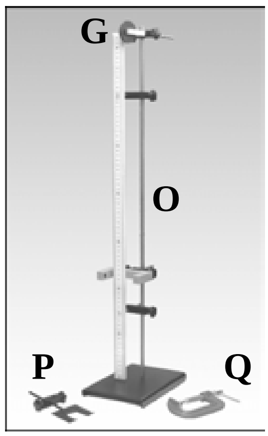
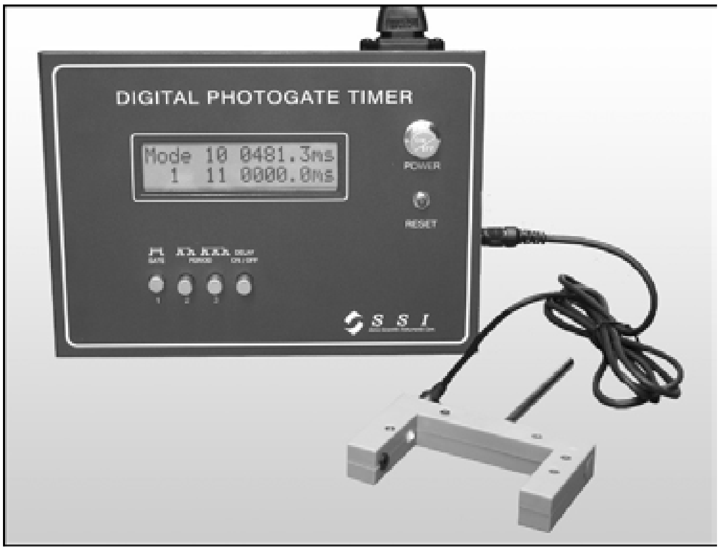


2. Instruction for the Photogate Timer

The Photogate consists of an infrared LED and a photodetector. By connecting the Photogate to the Photogate Timer, you can measure the time duration related to the blocking of the infrared light reaching the sensor.

- Be sure that the Photogate is connected to the Photogate Timer. Turn on the power by pushing the button labelled “POWER”.
- To measure the time duration of a single blocking event, push the button labelled “GATE”. Use this “GATE” mode for speed measurements.
- To measure the time interval between two or three successive blocking events, push the corresponding “PERIOD”. Use this “PERIOD” mode for oscillation measurements.
- If “DELAY” button is pushed in, the Photogate Timer displays the result of each measurement for 5 seconds and then resets itself.
- If “DELAY” button is pushed out, the Photogate Timer displays the result of the previous measurement until the next measurement is completed.
- After any change of button position, press the “RESET” button once to activate the mode change.

Caution: Do not look directly into the Photogate. The invisible infrared light may be harmful to your eyes.



Photogate, Photogate Timer, and connection cable

3. Instruction for the Electronic Balance

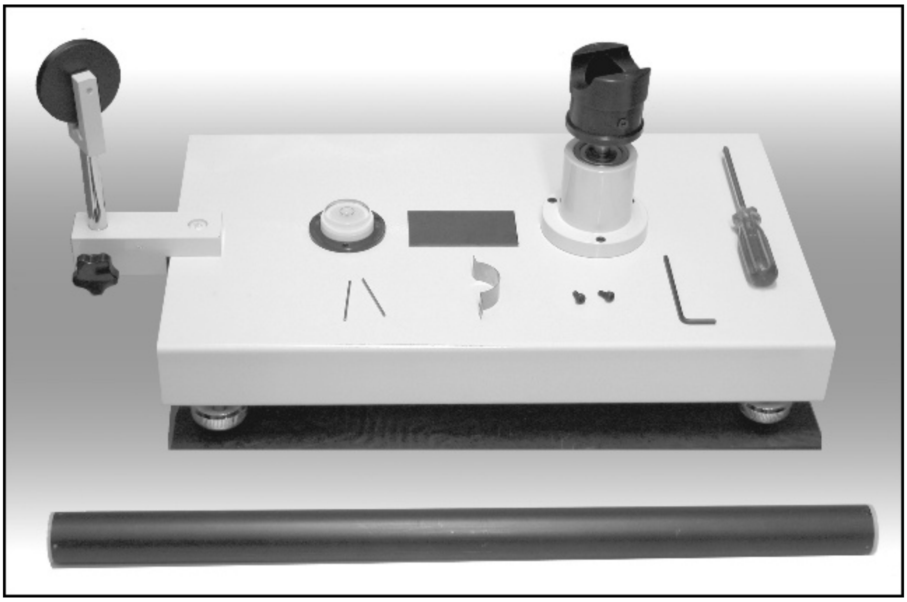
- Adjust the bottom legs to set the balance stable. (Although there is a level indicator, setting the balance in a completely horizontal position is not necessary.)
- Without putting anything on the balance, turn it on by pressing the "On/Off" button.
- Place an object on the round weighing pan. Its mass will be displayed in grams.
- If there is nothing on the weighing pan, the balance will be turned off automatically in about 25 seconds.

Balance

4. Instruction for the Rotation Stage

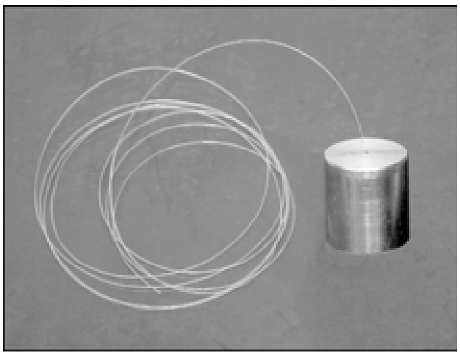
- Adjust the bottom legs to set the rotation stage stable on a rubber pad in a near horizontal position.
- With a U-shaped plate and two screws, mount the Mechanical "Black Box" (black cylinder) on the top of the rotating stub. Use Allen (hexagonal, L-shaped) wrench to tighten the screws.
- The string attached to the weight is to be fixed to the screw on the side of the rotating stub. Use the Philips screw driver.

Caution: Do not look too closely at the Mechanical "Black Box" while it is rotating.
Your eyes may get hurt.



Mechanical "Black Box" and rotation stage

Rotating stub



Weight with a string

Mechanical "Black Box"

[Question] ***Find the mass of the ball and the spring constants of two springs in the Mechanical "Black Box".***

General Information on the Mechanical "Black Box"

The Mechanical "Black Box" (MBB) consists of a solid ball attached to two springs in a black cylindrical tube as shown in Fig. 1. The two springs are fashioned from the same tightly wound spring with different number of turns. The masses and the lengths of the springs when they are not extended can be ignored. The tube is homogeneous and sealed with two identical end caps. The part of the end caps plugged into the tube is 5 mm long. The radius of the ball is 11 mm and the inner diameter of the tube is 23 mm. The gravitational acceleration is given as g = 9.8 m/s^2. There is a finite friction between the ball and the inner walls of the tube.

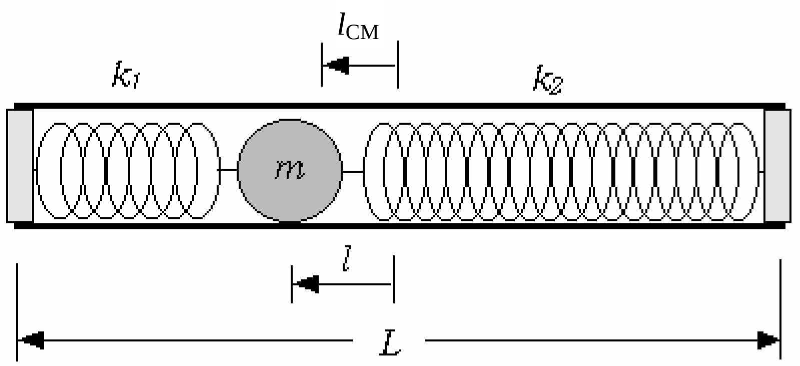


Fig. 1 Mechanical "Black Box" (not to scale)

The purpose of this experiment is to find out the mass m of the ball and the spring constants k_1 and k_2 of the springs without opening the MBB. The difficult aspect of this problem is that any single experiment cannot provide the mass m or the position l of the ball because the two quantities are interconnected. Here, l is the distance between the centers of the tube and the ball when the MBB lies horizontally in equilibrium when the friction is zero.

The symbols listed below should be used to represent the physical quantities of interest. If you need to use other physical quantities, use symbols different from those already assigned below to avoid confusion.

Assigned Physical Symbols

Mass of the ball: m

Radius of the ball: r (= 11 mm)

Mass of the MBB excluding the ball: M

Length of the black tube: L

Length of each end cap extending into the tube: δ (= 5.0 mm)

Distance from the center-of-mass of the MBB to the center of the tube: l_{CM}

Distance between the center of the ball and the center of the tube: x (or l at equilibrium when the MBB is horizontal)

Gravitational acceleration: g (= 9.8 m/s^2)

Mass of the weight attached to a string: m_o

Speed of the weight: v

Downward displacement of the weight: h

Radius of the rotating stub where the string is to be wound: R

Moments of inertia: I, I_o, I_1, I_2, and so on

Angular velocity and angular frequencies: ω, ω_1, ω_2, and so on

Periods of oscillation: T_1, T_2

Effective total spring constant: k

Spring constants of the two springs: k_1, k_2

Number of turns of the springs: N_1, N_2

Caution: Do not try to open the MBB. If you open it, you will be disqualified and your mark in the Experimental Competition will be zero.

Caution: Do not shake violently nor drop the MBB. The ball may be detached from the springs. If your MBB seems faulty, report to the proctors immediately. It will be replaced only once without affecting your mark. Any further replacement will cut down your mark by 0.5 points each time.

PART-A **Product of the mass and the position of the ball ($m \times l$)** (4.0 points)

l is the position of the center of the ball relative to that of the tube when the MBB lies horizontally in equilibrium as in Fig. 1. Find the value of the product of the mass m and the position l of the ball experimentally. You will need this to determine the value of m in ***PART-B***.

1. Suggest and justify, by using equations, a method allowing to obtain $m \times l$. (2.0 points)

2. Experimentally determine the value of $m \times l$. (2.0 points)

PART-B **The mass *m* of the ball** (10.0 points)

Figure 2 shows the MBB fixed horizontally on the rotating stub and a weight attached to one end of a string whose other end is wound on the rotating stub. When the weight falls, the string unwinds, and the MBB rotates. By combining the equation pertinent to this experiment with the one obtained in ***PART-A***, you can find an equation for *m*.

Between the ball and the inner walls of the cylindrical tube acts a frictional force. The physical mechanisms of the friction and the slipping of the ball under the rotational motion are complicated. To simplify the analysis, you may ignore the energy dissipation due to kinetic friction.

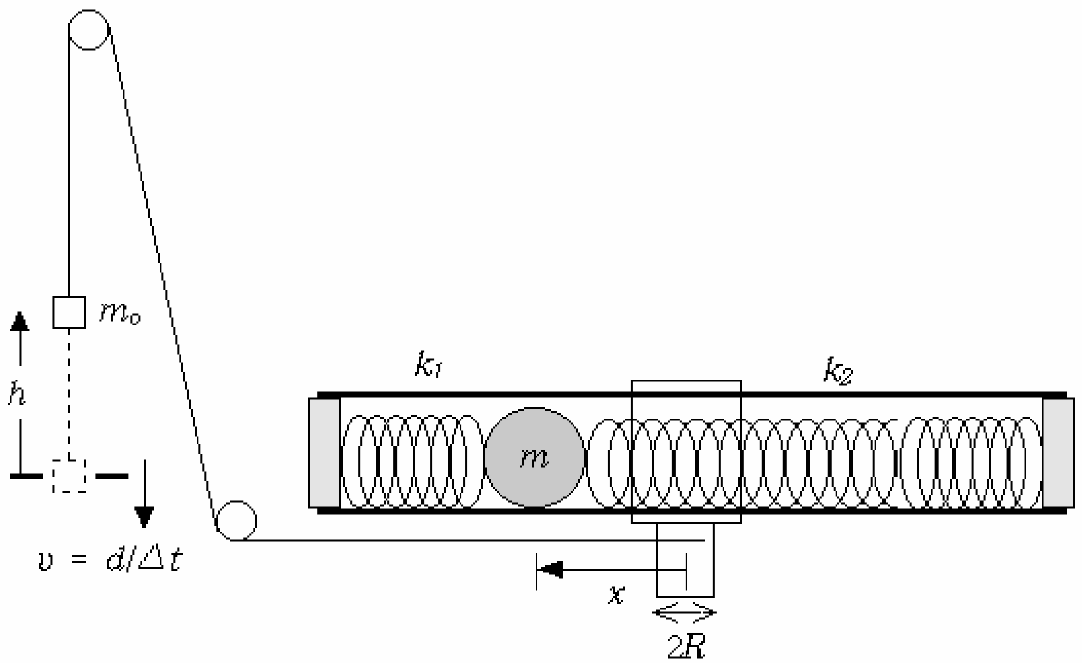


Fig. 2 Rotation of the Mechanical "Black Box" (not to scale)
The angular velocity ω of the MBB can be obtained from the speed v of the weight passing through the Photogate. x is the position of the ball relative to the rotation axis, and d is the length of the weight.

1. Measure the speed of the weight v for various values of downward displacement h of the weight. It is recommended to scan the whole range from $h = 1.0\times10^{-2}$ m to 4.0×10^{-1} m by measuring v just once at each h with an interval of 1.0×10^{-2} ~2.0×10^{-2} m. Plot the data on graph paper in a form that is suitable to find the value of m. After you get a general idea of the relation between v and h, you may repeat the measurement or add some data points, if necessary. When the MBB rotates slowly, the ball does not slip from its static equilibrium position because of the friction between the ball and the tube. When the MBB rotates sufficiently fast, the ball hits and actually stays at the end cap of the tube because the springs are weak. Identify the slow rotation region and the fast rotation region on the graph. (4.0 points)

2. Show your measurements are consistent with the fact that h is proportional to v^2 ($h = Cv^2$) in the slow rotation region. Show from your measurements that $h = Av^2+B$ in the fast rotation region. (1.0 points)

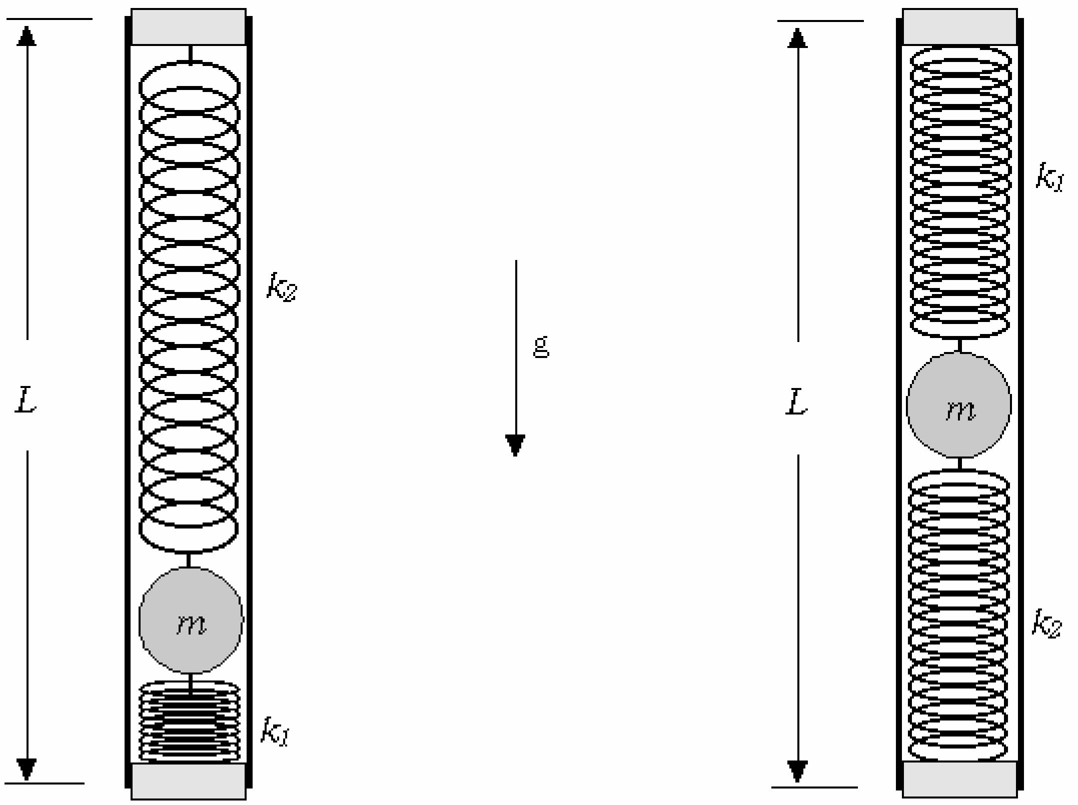
3. The moment of inertia of a ball of radius r and mass m about the axis passing through its center is $2mr^2/5$. If the ball is displaced a distance a perpendicular to the axis, the moment of inertia increases by ma^2. Use the symbol I to represent the total moment of inertia of all the rotating bodies excluding the ball. Relate the coefficient C to the parameters of the MBB such as m, l, etc. (1.0 points)

4. Relate the coefficients A and B to the parameters of the MBB such as m, l, etc. (1.0 points)

5. Determine the value of m from your measurements and the results obtained in ***PART-A***. (3.0 points)

PART-C **The spring constants k_1 *and* k_2** (6.0 points)

In this part, you need to perform small oscillation experiments using the MBB as a rigid pendulum. There are two small holes at each end of the MBB. Two thin pins inserted into the holes can be used as the pivot of small oscillation. The U-shaped support is to be clamped to the stand and used to support the pivot. Note that the angular frequency ω of small oscillation is given as ω = [torque/(moment of inertia × angle)]$^{1/2}$. Here, the torque and the moment of inertia are with respect to the pivot. Similarly to ***PART-B***, consider two experimental conditions, shown in Fig. 3, to avoid the unknown moment of inertia I_o of the MBB excluding the ball.



(1) (2)

Fig. 3 Oscillation of the Mechanical "Black Box" (not to scale)
The periods of small oscillation, T_1 and T_2, for two configurations shown above can be measured using the Photogate. Two pins and a U-shaped support are supplied for this experiment.

1. Measure the periods T_1 and T_2 of small oscillation shown in Figs 3(1) and (2) and write down their values, respectively. (1.0 points)

2. Explain (by using equations) why the angular frequencies ω_1 and ω_2 of small oscillation of the configurations are different. Use the symbol I_o to represent the moment of inertia of the MBB excluding the ball for the axis perpendicular to the MBB at the end. Use the symbol Δl as the displacement of the ball from the horizontal equilibrium position. (1.0 points)

3. Evaluate Δl by eliminating I_o from the previous results. (1.0 points)

4. By combining the results of ***PART-C*** 1~3 and ***PART-B***, find and write down the value of the effective total spring constant k of the two-spring system. (2.0 points)

5. Obtain the respective values of k_1 and k_2. Write down their values. (1.0 points)

Theoretical Question 1:

"Ping-Pong" Resistor

A capacitor consists of two circular parallel plates both with radius R separated by distance d, where $d << R$, as shown in Fig. 1.1(a). The top plate is connected to a *constant* voltage source at a potential V while the bottom plate is grounded. Then a thin and small disk of mass m with radius r ($<< R, d$) and thickness t ($<< r$) is placed on the center of the bottom plate, as shown in Fig. 1.1(b).

Let us assume that the space between the plates is in vacuum with the dielectric constant ε_0; the plates and the disk are made of perfect conductors; and all the electrostatic edge effects may be neglected. The inductance of the whole circuit and the relativistic effects can be safely disregarded. The image charge effect can also be neglected.

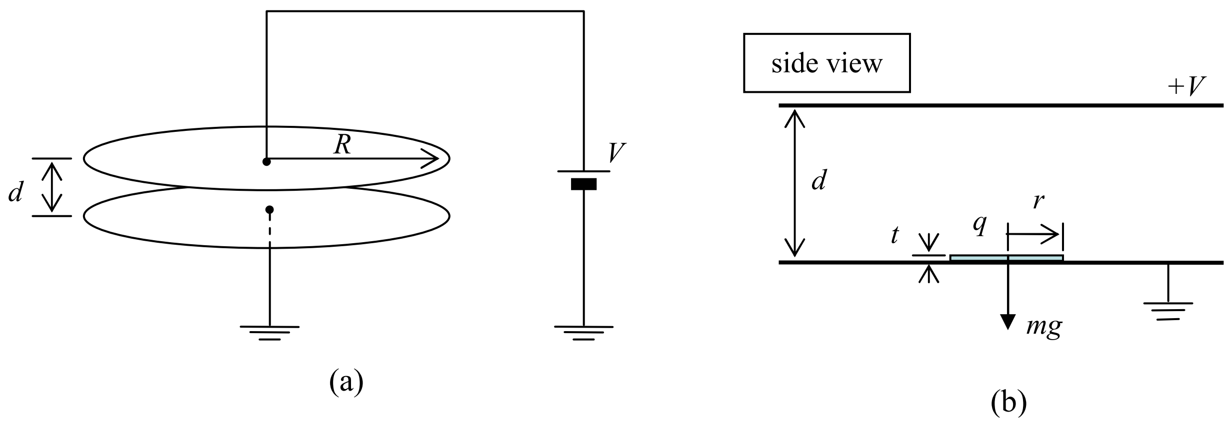


Figure 1.1 Schematic drawings of (a) a *parallel plate* capacitor connected to a constant voltage source and (b) a side view of the parallel *plates* with a small *disk* inserted inside the capacitor. (See text for details.)

(a) [1.2 points] Calculate the electrostatic force F_{p} between *the plates* separated by d before inserting the disk in-between as shown in Fig. 1.1(a).

(b) [0.8 points] When the disk is placed on the bottom plate, a charge q on *the disk* of Fig. 1.1(b) is related to the voltage V by $q = \chi V$. Find χ in terms of r, d, and ε_0.

(c) [0.5 points] The parallel plates lie perpendicular to a uniform gravitational field g. To lift up the disk at rest initially, we need to increase the applied voltage beyond a threshold voltage V_{th}. Obtain V_{th} in terms of m, g, d, and χ.

(d) [2.3 points] When $V > V_{\text{th}}$, the disk makes an up-and-down motion between the plates. (Assume that the disk moves only vertically *without any wobbling*.) The collisions between the disk and the plates are inelastic with the restitution coefficient $\eta \equiv (\text{v}_{\text{after}} / \text{v}_{\text{before}})$, where v_{before} and v_{after} are the speeds of the disk just before and after the collision respectively. The plates are stationarily fixed in position. The speed of the disk *just after* the collision at the bottom plate approaches a "steady-state speed" v_{s}, which depends on V as follows:

$$\text{v}_{\text{s}} = \sqrt{\alpha V^2 + \beta} \,. \tag{1.1}$$

Obtain the coefficients α and β in terms of m, g, χ, d, and η. Assume that the whole surface of the disk touches the plate evenly and simultaneously so that the complete charge exchange happens instantaneously at every collision.

(e) [2.2 points] After reaching its steady state, the time-averaged current I through the capacitor plates can be approximated by $I = \gamma V^2$ when $qV >> mgd$. Express the coefficient γ in terms of m, χ, d, and η.

(f) [3 points] When the applied voltage V is decreased (extremely slowly), there exists a critical voltage V_{c} below which the charge will cease to flow. Find V_{c} and the corresponding current I_{c} in terms of m, g, χ, d, and η. By comparing V_{c} with the lift-up threshold V_{th} discussed in (c), make a rough sketch of the $I-V$ characteristics when V is increased and decreased in the range from $V = 0$ to $3V_{\text{th}}$.

Theoretical Question 2

Rising Balloon

A rubber balloon filled with helium gas goes up high into the sky where the pressure and temperature decrease with height. In the following questions, assume that the shape of the balloon remains spherical regardless of the payload, and neglect the payload volume. Also assume that the temperature of the helium gas inside of the balloon is always the same as that of the ambient air, and treat all gases as ideal gases. The universal gas constant is R =8.31 J/mol·K and the molar masses of helium and air are $M_H = 4.00\times10^{-3}$ kg/mol and $M_A = 28.9\times10^{-3}$ kg/mol, respectively. The gravitational acceleration is $g = 9.8$ m/s^2.

[Part A]

(a) [1.5 points] Let the pressure of the ambient air be P and the temperature be T. The pressure inside of the balloon is higher than that of outside due to the surface tension of the balloon. The balloon contains n moles of helium gas and the pressure inside is $P+\Delta P$. Find the buoyant force F_B acting on the balloon as a function of P and ΔP.

(b) [2 points] On a particular summer day in Korea, the air temperature T at the height z from the sea level was found to be $T(z) = T_0(1 - z/z_0)$ in the range of $0 < z < 15$ km with z_0 =49 km and T_0 =303 K. The pressure and density at the sea level were P_0 = 1.0 atm = 1.01×10^5 Pa and ρ_0 = 1.16 kg/m^3, respectively. For this height range, the pressure takes the form

$$P(z) = P_0(1 - z/z_0)^\eta \ . \tag{2.1}$$

Express η in terms of z_0, ρ_0, P_0, and g, and find its numerical value to the *two* significant digits. Treat the gravitational acceleration as a constant, independent of height.

[Part B]

When a rubber balloon of spherical shape with un-stretched radius r_0 is inflated to a sphere of radius r $(\geq r_0)$, the balloon surface contains extra elastic energy due to the stretching. In a simplistic theory, the elastic energy at constant temperature T can be expressed by

$$U = 4\pi r_0^2 \kappa RT(2\lambda^2 + \frac{1}{\lambda^4} - 3) \quad (2.2)$$

where $\lambda \equiv r/r_0$ (≥ 1) is the size-inflation ratio and κ is a constant in units of mol/m^2.

(c) [2 points] Express ΔP in terms of parameters given in Eq. (2.2), and sketch ΔP as a function of $\lambda = r/r_0$.

(d) [1.5 points] The constant κ can be determined from the amount of the gas needed to inflate the balloon. At T_0=303 K and P_0=1.0 atm = 1.01×10^5 Pa, an un-stretched balloon ($\lambda = 1$) contains n_0=12.5 moles of helium. It takes n=3.6n_0=45 moles in total to inflate the balloon to λ=1.5 at the same T_0 and P_0. Express the balloon parameter a, defined as $a = \kappa/\kappa_0$, in terms of n, n_0, and λ, where $\kappa_0 \equiv \frac{r_0 P_0}{4RT_0}$. Evaluate a to the two significant digits.

[Part C]

A balloon is prepared as in (d) at the sea level (inflated to $\lambda = 1.5$ with $n = 3.6n_0 = 45$ moles of helium gas at T_0=303 K and P_0=1 atm=1.01×10^5 Pa). The total mass including gas, balloon itself, and other payloads is $M_T = 1.12$ kg. Now let the balloon rise from the sea level.

(e) [3 points] Suppose that the balloon eventually stops at the height z_f where the buoyant force balances the total weight. Find z_f and the inflation ratio λ_f at that height. Give the answers in two significant digits. Assume there are no drift effect and no gas leakage during the upward flight.

Theoretical Question 3

Atomic Probe Microscope

Atomic probe microscopes (APMs) are powerful tools in the field of nano-science. The motion of a cantilever in APM can be detected by a photo-detector monitoring the reflected laser beam, as shown in Fig. 3.1. The cantilever can move only in the vertical direction and its displacement z as a function of time t can be described by the equation

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F\,, \qquad (3.1)$$

where m is the cantilever mass, $k=m\omega_0^2$ is the spring constant of the cantilever, b is a small damping coefficient satisfying $\omega_0 >> (b/m) > 0$, and finally F is an external driving force of the piezoelectric tube.

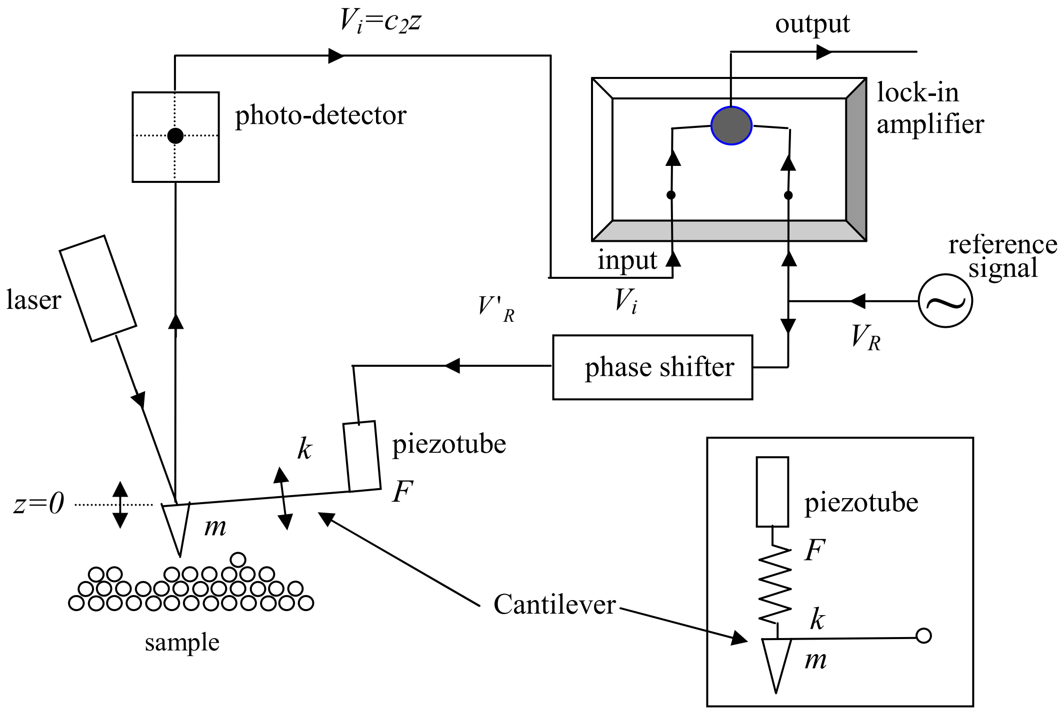


Figure 3.1 A schematic diagram for a scanning probe microscope (SPM). The inset in the lower right corner represents a simplified mechanical model to describe the coupling of the piezotube with the cantilever.

[Part A]

(a) [1.5 points] When $F=F_0\sin\omega t$, $z(t)$ satisfying Eq. (3.1) can be written as $z(t)=A\sin(\omega t-\phi)$, where $A>0$ and $0\le\phi\le\pi$. Find the expression of the amplitude A and $\tan\phi$ in terms of F_0, m, ω, ω_0, and b. Obtain A and the phase ϕ at the resonance frequency $\omega = \omega_0$.

(b) [1 point] A lock-in amplifier shown in Fig.3.1 multiplies an input signal by the lock-in reference signal, $V_R = V_{R0}\sin\omega t$, and then passes *only* the dc (direct current) component of the multiplied signal. Assume that the input signal is given by $V_i = V_{i0}\ \sin(\omega_i t - \phi_i)$. Here V_{R0}, V_{i0}, ω_i, and ϕ_i are all positive given constants. Find the condition on ω (>0) for a non-vanishing output signal. What is the expression for the magnitude of the non-vanishing *dc output signal* at this frequency?

(c) [1.5 points] Passing through the phase shifter, the lock-in reference voltage $V_R = V_{R0}\sin\omega t$ changes to $V'_R = V_{R0}\sin(\omega t + \pi/2)$. V'_R, applied to the piezoelectric tube, drives the cantilever with a force $F = c_1 V'_R$. Then, the photo-detector converts the displacement of the cantilever, z, into a voltage $V_i = c_2 z$. Here c_1 and c_2 are constants. Find the expression for the magnitude of the *dc output signal* at $\omega = \omega_0$.

(d) [2 points] The small change Δm of the cantilever mass shifts the resonance frequency by $\Delta\omega_0$. As a result, the phase ϕ at the original resonance frequency ω_0 shifts by $\Delta\phi$. Find the mass change Δm corresponding to the phase shift $\Delta\phi = \pi/1800$, which is a typical resolution in phase measurements. The physical parameters of the cantilever are given by $m = 1.0\times10^{-12}$ kg, $k = 1.0$ N/m, and $(b/m) = 1.0\times10^{3}$ s^{-1}. Use the approximations $(1+x)^a \approx 1 + ax$ and $\tan(\pi/2 + x) \approx -1/x$ when $|x| << 1$.

[Part B]

From now on let us consider the situation that some forces, besides the driving force discussed in Part A, act on the cantilever due to the sample as shown in Fig.3.1.

(e) [1.5 points] Assuming that the additional force $f(h)$ depends only on the distance h between the cantilever and the sample surface, one can find a new equilibrium position h_0. Near $h = h_0$, we can write $f(h) \approx f(h_0) + c_3(h - h_0)$, where c_3 is a constant in h. Find the new resonance frequency ω'_0 in terms of ω_0, m, and c_3.

(f) [2.5 points] While scanning the surface by moving the sample horizontally, the tip of the cantilever charged with $Q = 6e$ encounters an electron of charge $q = e$ trapped (localized in space) at some distance below the surface. During the scanning around the electron, the maximum shift of the resonance frequency $\Delta\omega_0 (= \omega'_0 - \omega_0)$ is observed to be much smaller than ω_0. Express the distance d_0 from the cantilever to the trapped electron at the maximum shift in terms of m, q, Q, ω_0, $\Delta\omega_0$, and the Coulomb constant k_e. Evaluate d_0 in nm (1 nm = 1×10^{-9} m) for $\Delta\omega_0 = 20$ s^{-1}.

The physical parameters of the cantilever are $m = 1.0\times10^{-12}$ kg and $k = 1.0$ N/m. Disregard any polarization effect in both the cantilever tip and the surface. Note that $k_e = 1/4\pi\varepsilon_0 = 9.0\times10^{9}$ N·m^2/C^2 and $e = -1.6\times10^{-19}$ C.